NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SUPERSONIC FLOW PAST TWO OSCILLATING AIRFOILS

by

Georgios Alexandris

June 1998

Thesis Advisor: M. F. Platzer
Co-Advisor: James Luscombe

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	June 1998	Master's Thesis

4. TITLE AND SUBTITLE **SUPERSONIC FLOW PAST TWO OSCILLATING AIRFOILS**	5. FUNDING NUMBERS
6. AUTHOR(S) Alexandris,Georgios	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for public release; distribution is unlimited.	

13. ABSTRACT

Supersonic flow past two oscillating airfoils is analyzed using an elementary analytical theory valid for low frequencies of oscillation. The airfoils may have arbitrary stagger angle. This approach generalizes Sauer' solution for a single airfoil oscillating at small frequencies in an unbounded supersonic flow.

It is shown that this generalization can provide an elementary theory for supersonic flow past two slowly oscillating airfoils. This aerodynamic tool will facilitate the evaluation of pressure distributions and consequently the calculation of moment coefficient. Torsional flutter boundaries are computed. The results for the pitch-damping coefficient are the same when compared with previous analysis.For arbitrary frequencies a linearized method of characteristics was outlined.

The elementary theory that has been developed in the thesis can be used for flutter evaluation of aicraft carrying external stores. The result of the thesis is the derivation of the pitch-damping coefficient, which is necessary to predict the flutter conditions.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Flutter analysis, Structures	86
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFI- CATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

Approved for public release; distribution is unlimited

SUPERSONIC FLOW PAST TWO OSCILLATING AIRFOILS

Georgios Alexandris
Major, Hellenic Air Force
B.S., Hellenic Air Force Academy, 1983

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN APPLIED PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1998

Author: ______________________
Georgios Alexandris

Approved by: ______________________
M. F.Platzer, Thesis Advisor

____
James Luscombe, Thesis Co-Advisor

____
S. Weber, Thesis Co-Advisor

____
B. Maier, Chairman, Department of Physics

ABSTRACT

Supersonic flow past two oscillating airfoils is analyzed in the thesis using an analytical elementary theory valid for low frequencies of oscillation. The airfoils may have arbitrary stagger angle. This approach generalizes Sauer's solution for a single airfoil oscillating at small frequencies in an unbounded supersonic flow.

It is shown that this generalization can provide an elementary theory for supersonic flow past two oscillating airfoils. This aerodynamic tool will facilitate the calculation of pressure distribution and consequently the calculation of moment coefficient. Torsional flutter boundaries are computed. The results for the pitch damping coefficient are the same when compared with previous analysis. For arbitrary frequencies a linearized method of characteristics was outlined.

The elementary theory that has been developed in the thesis can be used for flutter evaluation of aircraft carrying external stores. The result of the thesis is the derivation of the pitch damping coefficient which is necessary to predict the flutter conditions.

TABLE OF CONTENTS

TABLE OF SYMBOLS

A	h·cotα
B	h·tanβ
C	complex velocity of sound perturbation amplitude
c	velocity of sound perturbation
$\bar{c}$	local velocity of sound
c_∞	free stream velocity of sound
c	blade chord
cotα	$\sqrt{M^2-1}$
C_m	pitching moment coefficient
$c_{m\theta}$	pitching moment coefficient amplitude (real part)
$c_{m\theta}'$	pitching moment coefficient amplitude (imaginary part)
Cp,cp	pressure coefficient amplitude
C_θ	blade torsional stiffness
h	airfoil distance
g	structural damping
I_θ	airfoil moment of inertia

k	reduced frequency
L	aerodynamic lift
M	Mach number
$\mathbf{M_b}$	blade mass per unit span
$\mathbf{M_\theta}$	aerodynamic pitching moment
P	non dimensional complex pressure perturbation amplitude
p	pressure
$\mathbf{p_\infty}$	freestream pressure
$\mathbf{r_\theta}$	airfoil radius of gyration $\sqrt{\dfrac{4I_\theta}{M_b c^2}}$
R	universal gas constant
s	entropy
$\mathbf{S_\theta}$	airfoil static moment about the elastic axis per unit span
T	temperature
U	complex streamwise velocity perturbation amplitude
$\mathbf{U_F}$	freestream flutter speed
u	streamwise velocity perturbation
u	internal energy
$\bar{\mathbf{u}}$	local streamwise velocity
$\mathbf{u_\infty}$	freestream velocity
V	complex normal velocity perturbation amplitude

v	normal velocity perturbation
X	frequency rate $\left[\frac{\omega_\theta}{\omega}\right]^2$
x_θ	distance between airfoil center of gravity and elastic axis
x_0	elastic axis position
α	Mach angle
β	stagger angle
γ	ratio of specific heats
δ	phase angle
Φ	complex velocity perturbation potential
φ	complex velocity perturbation potential amplitude
θ	angle of attack
θ_0	amplitude of pitch oscillations
ω	frequency of oscillations
ω_F	flutter frequency
ω_θ	torsional natural frequency
Ω_θ	factor μr_θ^2
Ω	factor $\mu \frac{\omega_h^2}{\omega_\theta^2}$
ρ	local density
ρ_∞	freestream density
ξ,η	characteristics coordinate

I. INTRODUCTION

Aeroelasticity is the study of the effect of aerodynamic forces on elastic bodies. One of the interesting problems in aeroelasticity is the stability of a structure in wind. Since for a given configuration of the elastic body, the aerodynamic force increases rapidly with the wind speed while the elastic stiffness is independent of the wind, there may exist a critical wind speed at which the structure becomes unstable. Such instability could create excessive deformation and failure of the structure. A particular problem is the FLUTTER of structures, which is a self-excited vibration phenomenon.

Flutter is caused by the interaction of all three forces: aerodynamic, elastic and inertial. Consider as an example a cantilever wing mounted in a wind tunnel (with the root rigidly built in). Suppose the wing is deliberately deflected and then released. At sufficiently low wind speed, the oscillation that follows this disturbance will quickly die out. However, at some higher wind speed one observes a steady oscillation, which maintains itself. This is the critical flutter speed for the wing. At higher speeds, the oscillation will be rapidly divergent causing structural failure.

When large external bodies such as engine nacelles, fuel tanks, or electronic warfare pods are added to the wing of an aircraft, the dynamic characteristics of an aircraft will be changed. Over the last several decades, there has been considerable interest in the calculation of the unsteady aerodynamic forces on aircraft. Many of the methods presently in use for numerical calculations involve application of a two-dimensional theory to solve three-dimensional flow problems. Most of these methods do not have the ability to model an arbitrarily shaped aircraft. However, with certain assumptions, it is possible to simplify the flow problems to adequately account for the effects of unsteady aerodynamic forces on aircraft components. Unfortunately, the transonic flight regime still makes it very difficult to determine the unsteady aerodynamic forces because of the nonlinear shock and flow separation effects, which occur in this flight regime.

A good example for the requirement of the wing/stores flutter analysis is the fighter aircraft.

Fighter aircraft are commonly designed for a primary mission, such as air superiority, which may require few, if any, wing-mounted external stores. However, for increased effectiveness and versatility, many secondary missions evolve which are necessary and require the use of a variety of external stores. Thus, many combinations of these external stores must be carried at various stations on the wings to achieve the complex, multi-role missions required by the operational commands. Sometimes, the wing stores are not available to the aircraft manufacturer even during the production of an aircraft and in that case, the flutter analysis should be conducted using data not available from the original design.

A typical modern tactical fighter can carry a great variety of wing stores and consequently the total number of possible aircraft/store configurations is huge. Therefore, on military aircraft the effect of unsteady aerodynamics is computed for both the clean wings (no stores) and the wing with tip missiles. Using these two aerodynamic configurations, the flutter analysis is probably done on between 300 and 400 selected wing/store configurations. The cost for the above effort would increase by several orders of magnitude if wing/store aerodynamics were also considered for each of these configurations. In many cases, this is not necessary since the store aerodynamics has a small effect on the flutter speed. However, there are cases where neglecting the store aerodynamics will lead to the overestimation of the flutter speed. Therefore, the determination of the cost effectiveness of such analysis is a critical step.

In this thesis, we first review the most important papers, which have been published in recent years to analyze wing-store flutter problems. It is evident from the literature survey that most approaches are based on modern CFD (Computational Fluid Dynamics) approaches which require extensive computations.

However, there is still some merit in simplified analyses, which reveals the major physical effects. Limiting our approach to low supersonic flight speed we propose to replace the multiple stores underneath the wing by a second wing and to apply to a 2-D problem the so called " strip theory approach" which reduces the three dimensional flow problem. This simplification has the advantage of accounting for the interference effects between two lifting surfaces. Moreover, these interference effects will be stronger than those caused by the actual stores. The insights which can be attained with such an analysis, therefore will represent an extreme limiting case, but it will nevertheless reveal important physical effects.

The importance of such interference effects is well known in transonic and supersonic wind tunnel testing and in the operation of transonic and supersonic compressors. Platzer (1973) presented an analytical theory for the analysis of oscillatory supersonic wind tunnel and blade interference effects. This theory is based on the assumption that the actual airfoil or compressor blade can be replaced by a flat plate and that it oscillates with low frequency. This makes it possible to expand the solution in powers of the reduced frequency and to retain only the zeroth and first order term. Platzer (1973) showed that this theory provides a convenient analytical way to estimate the pitch damping as a function of supersonic Mach number and pitch-axis (elastic axis) position.

In this thesis, we modify Platzer's solution to the case of two airfoils, which are in close proximity to each other. Two cases need to be distinguished, namely the one with supersonic leading edge locus and the case with subsonic leading edge locus. These two cases and the basic theory will be explained in chapter III. The extension of Platzer's theory will be given in chapter IV. This is followed by a chapter on the method of characteristic approach, which can be used to compare with the analytical results. The final chapter is devoted to the flutter analysis.

II. LITERATURE SURVEY

EVALUATION OF METHODS FOR PREDICTION AND PREVENTION OF WING/STORE FLUTTER.

Author(s): Pollock, S. J. Sotomayer, W. A.; Huttsell, L. J.; Cooley, D. E

Source: Collect Tech Pap AIAA ASME ASCE AHS Struct. Dyn. Mater. Conf. 22nd AIAA Dyn. Spec. Conf., April 6-10 1981, Atlanta, GA, New York, NY, pp. 362-372

In response to the need to reduce costs and improve safety for flutter evaluation of aircraft carrying external stores, the Flight Dynamics Laboratory (FDL) has sponsored several efforts in the technical areas of unsteady aerodynamics, flutter prediction, and active flutter suppression. This paper discusses each of these three areas as they relate to wing/store flutter and presents specific examples from analyses and tests. Steady and unsteady pressure measurements were obtained in a wind tunnel at subsonic, transonic and supersonic speeds on a fighter wing, tip-mounted launcher and store, and underwing pylon and store. Store flutter calculations were performed using both calculated and measured data to determine the influence of store aerodynamics on the flutter characteristics. To improve the accuracy and reduce the time and costs of flutter evaluations on the many store configurations carried by Air Force fighters, the Flight Dynamics Laboratory (FDL) has sponsored several programs in the technical areas of unsteady aerodynamics and flutter predictions. Also, the FDL has been sponsoring several efforts to explore the potential of active flutter suppression systems using feedback control techniques to provide the required stability and to avoid speed placards.

This paper reports on some of the FDL research related to wing/store flutter prediction and prevention. The research includes an unsteady aerodynamic measurement program for a representative fighter wing, with and without tip missile and underwing store with test data covering the Mach number range 0.6 to 1.35. Flutter analyses based on an FDL computer program specifically for use on aircraft with external stores are also described. Flutter trends using this computer program are presented for the wing with and without stores based on sectional force coefficients from wind tunnel measurements and from theoretical calculations. A brief description is given of some FDL programs in active flutter suppression,

and typical results are presented for wings with stores, which indicate significant potential for improvement in flutter speeds.

Flutter trends for three different fighter wing/ store configurations were calculated using the FACES flutter analysis procedure. Use was made of available modal vibration data and measured aerodynamic data. The predicted Mach number trends gave minimum flutter speeds at transonic and low supersonic speeds as would be expected from the trend of measured center of pressure and lift-curve slope data. Although the aerodynamics on the tip launcher had a somewhat detrimental effect on flutter, the aerodynamics on the tip store had a much larger detrimental effect. For the underwing store, the aerodynamic effects were beneficial for flutter since the pylon had the effect of decreasing the aerodynamic loading on the outer portion of the wing.

WIND TUNNEL TESTS ON A FIGHTER AIRCRAFT WING/STORE FLUTTER SUPPRESSION SYSTEM

Author(s): Turner, M. R.

Source: AGARD, Rep. on a Coop. Program on Active Flutter Suppression, Paper Presented at the Structural and Material Panel Meet., 50th April 1980, Athens, Greece AGARD.

A special system designed using analytical data was tested on a YF- 17 aircraft model in the NASA Langley 16' wind tunnel and succeeded in meeting the requirement to increase the flutter dynamic pressure by 70% at Mach equals 0. 8. The system was designed using a novel procedure, which provides these stability margins, uses minimum control surface movement in turbulence, and can be designed using either analytical or empirical data. Two wing tip accelerometers and a leading edge control surface were used. Empirical open loop transfer functions obtained during the test showed that the analytical data overestimated the response of the flutter mode to leading edge control surface excitation.

AERODYNAMICS USING TIME-LINEARIZATION TRANSONIC FLUTTER ANALYSIS

Author(s): Wong, Y.S., Lee, B.H.K.; Murty, H.S.

Source: Journal of Aircraft, v. 30, January-February 1993, pp. 144-145

A survey of the progress made in the development of numerical simulation techniques for unsteady transonic flow calculations are presented. Computational methods in three-dimensional unsteady transonic flows concentrate mainly on the transonic small disturbance equation and time-lineraization approach. An algorithm is introduced for solving flutter occurrence.

WING FLUTTER BOUNDARY PREDICTION USING UNSTEADY EULER AERODYNAMIC METHOD

Author(s): Lee-Rausch, Elizabeth M.; Batina, John T.

Source: Journal of Aircraft, v. 32, 2 March-April 1995, AIAA, Washington, DC, USA, pp. 416-422

Modifications to an existing three-dimensional, implicit, Euler/Reynolds-averaged Navier-Stokes code for the aeroelastic analysis of wings are described. These modifications include the incorporation of a deforming mesh algorithm and the addition of the structural equations of motion for their simultaneous time-integration with the governing flow equations. This article gives a brief description of these modifications and presents unsteady calculations that check the modifications to the code. Euler flutter results for an isolated 45-deg swept-back wing are compared with experimental data for seven freestream Mach numbers that define the flutter boundary over a range of Mach number from 0.499 to 1.14. These comparisons show good agreement in flutter characteristics for freestream Mach numbers below unity. For freestream Mach numbers above unity, the computed aeroelastic results predict a premature rise in the flutter boundary as compared with the experimental boundary. Steady and unsteady contours of surface Mach number and pressure are included to illustrate the basic flow characteristics of the time-marching flutter calculations and to aid in identifying possible causes for the increase in the computational flutter boundary.

APPLICATION OF TRANSONIC SMALL DISTURBANCE THEORY TO THE ACTIVE FLEXIBLE WING MODEL

Author(s): Silva, Walter A.; Bennett, Robert M

Source: Journal of Aircraft, v. 32, January-February 1995, AIAA, Washington, DC, USA, pp. 16-22

A code, developed at the NASA Langley Research Center, is applied to the active flexible wing wind-tunnel model for prediction of transonic aeroelastic behavior. A semispan computational model is used for evaluation of symmetric motions, and a full-span model is used for evaluation of antisymmetric motions. Static aeroelastic solutions using the computational aeroelasticity program-transonic small disturbance are computed. Dynamic (flutter) analyses are then performed as perturbations about the static aeroelastic deformations and presented as flutter boundaries in terms of Mach number and dynamic pressure. Flutter boundaries that take into account modal refinements, vorticity and entropy corrections, antisymmetric motions, and sensitivity to the modeling of the wingtip ballast stores are also presented and compared with experimental flutter results.

WING-STORE FLUTTER ANALYSIS OF AN AIRFOIL IN INCOMPRESSIBLE FLOW

Author(s): Yang, Zhi-Chun Zhao, Ling-Cheng

Source: Journal of Aircraft, v. 26, 6 June 1989, pp. 583-587

The flutter of two-dimensional airfoil with external store is analyzed to investigate the effects of pylon stiffness on flutter speed. Among the 40 configurations studied, five were tested in the wind tunnel to verify the analytical results. The variations of wing-store flutter speed with the pylon stiffness can be divided into three types. The curves of the normal and flutter frequencies vs pylon stiffness have the same pattern. They can be sketched approximately by the aid of the normal frequencies of the two degenerated two-degree-of-freedom systems, i.e., and the freely hinged and rigidly connected store cases. A limiting flutter speed for very small pylon stiffness is deduced, which is useful to identify which type of flutter the configuration studied belongs to.

STUDY OF THE EFFECT OF STORE AERODYNAMICS ON WĪNG/STORE FLUTTER.

Author(s): Turner, C. D.

Source: Collect Tech. Pap. AIAA ASME ASCE AHS Struct. Struct. Dyn. Mater. Conf. 22nd AIAA Dyn. Spec. Conf., April 6-10, 1981, and 1981, Atlanta, GA, AIAA (CP 812), New York, NY, pp. 343-351, Paper: 81-0604

This study represents the first systematic analytical study of the effect of store aerodynamics on wing/store flutter. A large number of wing/store single carriage configurations and parameters were included in the study; multivariate analysis techniques were used for the first time to analyze wing/store configurations, modal data, and flutter results. The results of the multivariate analysis indicate that it may not be possible to develop general guidelines, but it is possible to develop specific guidelines for use with a particular aircraft. This study was the first attempt to do a systematic analytical study of the effect of store aerodynamics on wing/store flutter. To determine this effect flutter analyses were done on four aircraft with single carriage of three basic store types. In all 308 configurations were analyzed with and without store aerodynamics (tip missiles, tip tanks, and underwing stores). The effects of stores have been analyzed. The results of the factor analysis indicate that it may not be possible to develop general guidelines, but it is possible to develop specific guidelines for use with a particular aircraft. The conclusions of the study, as far as it concerns the effect of store aerodynamics on wing-store flutter, show that 60% of the tested configurations require a change in the flutter speed no more than 7%. Therefore, there is no need to evaluate the requirements for aerodynamic modeling of the store. However, for the rest of the configurations there is such a need. Moreover, 75% of the last reveal that nonrealistic flutter results are obtained without taking into consideration the store aerodynamic. Of course, the results of this study represent the configurations that were used to generate the data base.

AERODYNAMIC MODELING OF AN OSCILLATING WING WITH EXTERNAL STORES

Author(s):, Sotomayer W. A., Dusto A. R, Epton M.A., Johnson F. T.

Source: AIAA , New York, NY, pp. 243-252, Paper: 81-0609

An analysis of the steady and unsteady aerodynamic forces acting on a fighter aircraft wing with stores has been done. Computations were performed with paneling methods capable of presenting arbitrary aircraft configurations in subsonic and supersonic flow. Interference effects from a tip store and an underwing pylon store in varying stages of completion were also analyzed. Detailed comparisons between experimental data and numerical computations are also made.

In order to calculate the pressures, forces and moments on an aircraft a numerical approach is proposed. Numerical computations are based on solving an integral equation formulation of the flow problem being considered. Steady state calculations were done with two numerical methods. The first of these is a pilot code developed by Johns. In this method, distributions of linearly varying sources and quadratically varying doublets are used to represent the aerodynamic surfaces. Wakes are represented by doublets with constant streamwise strength and linear variation in the spanwise direction. A method developed by Woodward was also employed in making numerical calculations. This method utilizes distributions of sources and vortices to represent an aerodynamic surface and the wakes shed from various components.

For unsteady flow, each of the methods makes use of sources and doublets as the basic aerodynamic singularities. In a method developed by Johnson for unsteady subsonic flow, distributions of sources and doublets are used to represent the aerodynamic surfaces. In the wake, doublets of fixed strength and position are used to represent the effects of unsteady wake motion. Calculations were also done with a special doublet lattice method. In this method, a wing is represented as a sheet of doublets, components such as the fuselage or a

store or nacelle can be represented with an axisymmetric distribution of sources with doublets on the surface.

Approximations are imposed on these distributions and on the kernel functions used to represent their influences on the flow. These approximations are briefly described along with the flow boundary conditions imposed on the aerodynamic surfaces and wakes.

It is interesting that the small disturbance partial differential equation is being used for unsteady inviscid incompressible flow along with the linearized form expressed in terms of the velocity potential φ. In addition, simple harmonic motion is assumed as in the thesis.

$\varphi=\mathrm{Re}[\varphi^* e^{i\delta}]$ where φ^* is a complex quantity.

Moreover using the Helmoltz equation a more simple relationship is established for φ^* with the form of an integral equation. An approximate solution of the last equation permits an evaluation of the entire flowfield. Numerical solutions of the last equation are achieved through an aerodynamic influence coefficient method. The boundary conditions are established taking into consideration the configuration of the F-5 aircraft, a relatively old aircraft. The aircraft is comprised of the following components:

1) wing
2) underwing pylon
3) missile rack
4) missile body.

As in the thesis, the above surfaces are approximated as infinitesimally thin surfaces so the thin wing theory applies except the for missile body. The boundary conditions are expressed in terms of Taylor series expansion about reference surfaces taking into consideration the thickness.

Initially boundary conditions for steady state flow are made use of and unsteady boundary conditions are taken as a perturbation about a steady state condition. In addition, variations in Mach number, frequency of oscillations, and interference effects arising from component build up are presented.

For the wing missile, the boundary conditions are of mixed type (Dirichlet and Newman). On the surface of the missile, the boundary conditions are similar to the other parts of the aircraft.

Using a Woodward method, a lifting surface is divided into numbers of aerodynamic panels number, containing distributions of sources and vortices. A source distribution represents a fuselage [or a pod or external store]. Wing thickness is represented by a linearly varying source distribution in which the strength is equated to the chordwise slope of the wing thickness. Camber, twist, and lifting effects are represented by a linearly varying vortex distribution where the strength is determined to satisfy tangential flow at panel control points. An iterative procedure is employed in solving the boundary value problem. For analysis problems, the surface slope is described, and singularity strengths are determined by inverting the matrix of aerodynamic influence coefficients. With the strengths of the aerodynamic singularities known, the u, v, and w velocity components at a given point may then be determined. Pressures, forces, and moments are calculated by numerical integration.

In the doublet lattice method, the aerodynamic surface is subdivided into a series of infinitesimally thin panels. Along the quarter chord line of each of these panels is contained a distribution of acceleration potential doublets. The strength of the distribution is constant but is not known. Specification of the normal velocity at a set of points on the surface determines the loading on each element. Locating the lifting elements at the quarter chord and the collocation point of the three-quarter chord of the midspan of each panel usually results in reasonable success. An approximation for the lifting pressure coefficients in terms of the induced normal velocities may then be found. Lift, moment, roll and generalized forces may then be calculated.

Recently, a series of wind tunnel tests were conducted by NLR of the Netherlands with sponsorship from the Air Force. These tests involved measurements of steady and unsteady pressures and forces on a model of the F-5 wing with external stores. Several external store arrangements, which are described and shown in this paper, are compared with numerical calculations. Experimental results and numerical calculations show that:

1) For subsonic flow, the interference due to the tip store is much greater on the outer portion of the wing than it is on the inner portions.

2) The interference effect due to the pylon store is significant on the lower surface of the wing for subsonic flow; there are also noticeable effects on the upper surface of the wing at subsonic speeds.

3) For supersonic speeds, the zone of influence remains localized within the Mach cone.

In general, numerical calculations and experimental results agreed reasonably well with each other.

III. PROBLEM FORMULATION

The interference between the airfoil (wing airfoil) and the wing stores is examined by making certain assumptions that simplify the problem. The wing airfoil is replaced by an oscillating flat plate airfoil and the wing stores by another oscillating flat plate in distance d from the wing airfoil. Two cases need to be distinguished, namely:

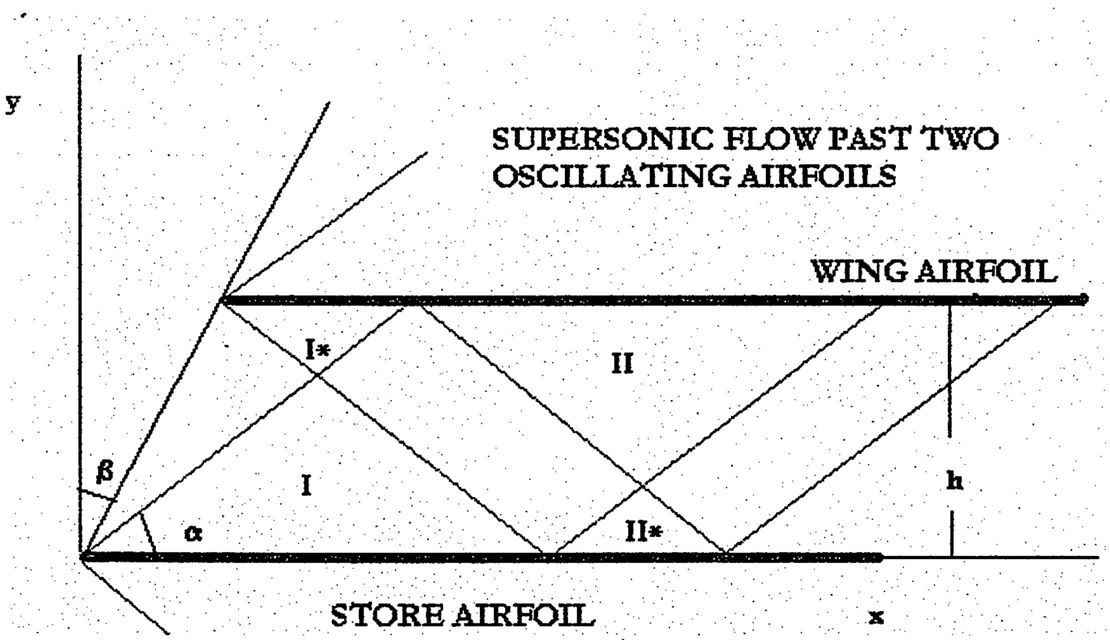

Fig. 1

1) Supersonic flow past two oscillating airfoils with supersonic leading edge locus shown in Fig. 1 (above)

2) Supersonic flow past two oscillating airfoils with subsonic leading edge locus shown in Fig. 2

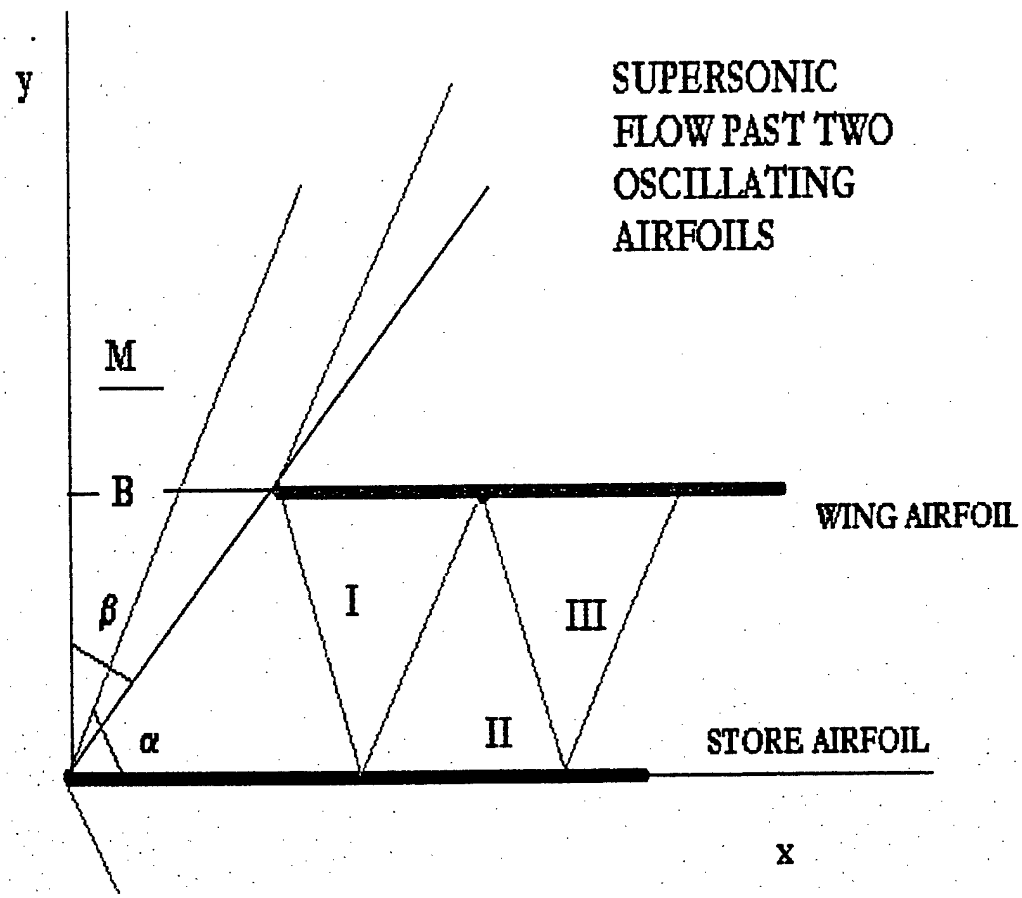
SUPERSONIC
FLOW PAST TWO
OSCILLATING
AIRFOILS
y
M
B
β
α
I
II
III
WING AIRFOIL
STORE AIRFOIL
x

Fig. 2

The differences are obvious and will be further analyzed during the examination of each case. In both cases multiple reflections occur. As explained in the preface the interference between the two airfoils under certain conditions could result in unsteady flow phenomena like flutter.

Therefore, the study of the oscillating airfoils is important in predicting the flutter characteristics or dynamic response of the wing airfoil when different stores are attached. For the case shown in Fig. 1:

$$\tan\beta < \cot\alpha \qquad \text{(III-1a)}$$

Where the case shown in Fig. 2 requires:

$$\tan\beta > \cot\alpha \qquad \text{(III-1b)}$$

The flow is assumed a non-viscous compressible two-dimensional flow of a perfect gas governed by the continuity equation, the Euler, and the energy equation. The continuity equation is:

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad \text{(III-2)}$$

The Euler Equations:

$$\frac{D\bar{u}}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \qquad \text{(III-3)}$$

$$\frac{Dv}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad \text{(III-4)}$$

The Energy Equation:

$$\frac{Ds}{Dt} = 0 \qquad \text{(III-5)}$$

Because of the assumption of small amplitude oscillations of the airfoils, all flow quantities are considered small perturbations linearly superimposed on free stream quantities. The velocities can therefore be written as:

$$\bar{u} = u_\infty + u \tag{III-6}$$

$$\bar{c} = c_\infty + c \tag{III-7}$$

$$v = v \tag{III-8}$$

In addition, the pressure and density perturbations are linearly superimposed on freestream quantities. Therefore:

$$\Delta p = p - p_\infty \tag{III-9}$$

$$\Delta \rho = \rho - \rho_\infty \tag{III-10}$$

The local velocity of sound is given by:

$$\bar{c} = \left(\frac{dp}{d\rho}\right)^2 S{=}ct \tag{III-11}$$

Furthermore:

$$\frac{p}{\rho^\gamma} = const \tag{III-12}$$

Taking the total differential:

$$\frac{1}{\rho^\gamma} dp - \gamma p \frac{1}{\rho^{\gamma+1}} d\rho = 0 \tag{III-13}$$

Or:

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho} \qquad \text{(III-14)}$$

So:

$$\overline{c^2} = \gamma \frac{p}{\rho} \qquad \text{(III-15)}$$

From AA 4318 class notes, the surface boundary conditions will be calculated. Requirement is that no flow will penetrate the solid surface of the airfoil resulting in the flow tangency condition:

$$\left(\overline{V} - \overline{V}_s\right) \cdot \overline{n} = 0 \qquad \text{(III-16)}$$

where V, V_s are the velocities of the free stream and the surface over which the fluid flows.If the body is described by the equation:

$$F(x, y, t) = 0 \qquad \text{(III-17)}$$

then the normal to the surface at any point is given by the following relation:

$$\overline{n} = \frac{\nabla F}{|\nabla F|} \qquad \text{(III-18)}$$

Taking into consideration that the substantial derivative of a surface particle should be zero:

$$\frac{\partial F}{\partial t} + \overline{V}_s \cdot \nabla F = 0 \qquad \text{(III-19)}$$

one obtains from (II-16):

$$\overline{V} \cdot \nabla F = \overline{V}_s \cdot \nabla F \qquad \text{(III-20)}$$

and therefore:

$$\frac{DF}{Dt}=\frac{\partial F}{\partial t}+\overline{V}\cdot\nabla F=0 \qquad \text{(III-21)}$$

Or for two dimensional flow:

$$\frac{DF}{Dt}=\frac{\partial F}{\partial t}+u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}=0 \qquad \text{(III-22)}$$

Particles in contact with the surface must have the same normal velocities as the surface. This means that the rate of change of F is zero hence:

$$\frac{DF}{Dt}=0 \qquad \text{(III-23)}$$

For an airfoil the equation of the upper surface can be written as follow:

$$F_u(x,y,t)=y-y_u(x,t)=0 \qquad \text{(III-24)}$$

Where y_u is the distance from the chord line to the upper surface. The equation of the lower surface can be written in a similar way:

$$F_L(x,y,t)=y-y_L(x,t)=0 \qquad \text{(III-25)}$$

At $y=y_u$:

$$\frac{DF_u}{Dt}=-\frac{\partial y_u}{\partial t}-(\bar{u})\cdot\frac{\partial y_u}{\partial x}+v=0 \qquad \text{(III-26)}$$

At $y=y_L$:

$$\frac{DF_L}{Dt}=-\frac{\partial y_L}{\partial t}-(\bar{u})\cdot\frac{\partial y_L}{\partial x}+v=0 \qquad \text{(III-27)}$$

$$\frac{\partial y_U}{\partial y}=\frac{\partial y_L}{\partial y}=1 \qquad \text{(III-28)}$$

Thus, the normal flow velocity can be written:

$$v = \frac{\partial y_u}{\partial t} + \bar{u}\frac{\partial y_u}{\partial x} \qquad \text{(III-29)}$$

Applying the assumption of linear perturbation theory one has $\bar{u} = u_\infty + u$, therefore:

$$v = \frac{\partial y_u}{\partial t} + u_\infty \frac{\partial y_u}{\partial x} + u\frac{\partial y_u}{\partial x}, .. y = y_u(x,t) \qquad \text{(III-30)}$$

$$v = \frac{\partial y_L}{\partial t} + u_\infty \frac{\partial y_L}{\partial x} + u\frac{\partial y_L}{\partial x}, .. y = y_L(x,t) \qquad \text{(III-31)}$$

Neglecting the higher order terms one obtains:

$$v = \frac{\partial y_u}{\partial t} + u_\infty \frac{\partial y_u}{\partial x}, y = y_u(x,t) \qquad \text{(III-32)}$$

$$v = \frac{\partial y_L}{\partial t} + u_\infty \frac{\partial y_L}{\partial x}, y = y_L(x,t) \qquad \text{(III-33)}$$

This normal velocity will be expanded as Taylor series of the normal flow velocity around y=0:

$$v(x, y_L, t) = v(x, 0^+, t) + y_L \frac{\partial v(x, 0^-, t)}{\partial y} + \frac{y_L{}^2}{2!}\frac{\partial^2 (x, 0^-, t)}{\partial^2 y} + ... \qquad \text{(III-34)}$$

$$v(x, y_L, t) = v(x, 0^+, t) + y_L \frac{\partial v(x, 0^-, t)}{\partial y} + \frac{y_L{}^2}{2!}\frac{\partial^2 (x, 0^-, t)}{\partial^2 y} + ... \qquad \text{(III-35)}$$

Using the assumption of thin airfoil and small linear perturbation theory the higher order terms may be neglected and one obtains:

$$v(x, y_u, t) = v(x, 0^+, t) \qquad \text{(III-36)}$$

$$v(x, y_L, t) = v(x, 0^-, t) \qquad \text{(III-37)}$$

Therefore:

$$v(x, 0^+, t) = \frac{\partial y_u}{\partial t} + u_\infty \frac{\partial y_u}{\partial x}, .. y = 0^+ \qquad \text{(III-38)}$$

$$v(x, 0^-, t) = \frac{\partial y_L}{\partial t} + u_\infty \frac{\partial y_L}{\partial x}, y = 0^- \qquad \text{(III-39)}$$

For simple harmonic motion of the airfoil:

$$y_u = h_u(x) e^{i\omega t} \qquad \text{(III-40)}$$

$$y_L = h_L(x) e^{i\omega t} \qquad \text{(III-41)}$$

Therefore, the linearized flow tangency equations can be written:

$$v(x, 0^+, t) = [i\omega h_u(x) + u_\infty \frac{\partial h_u}{\partial x}] e^{i\omega t}, .. y = 0^+ \qquad \text{(III-42)}$$

$$v(x, 0^-, t) = [i\omega y_L(x) + u_\infty \frac{\partial h_L}{\partial x}] e^{i\omega t}, .. y = 0^- \qquad \text{(III-43)}$$

The above relationships describe the normal velocity on the lower airfoil (store airfoil). Similarly, one obtains for the upper airfoil (wing airfoil) at y=d:

$$v(x, d^-, t) = [i\omega h_L(x) + u_\infty \frac{\partial h_L}{\partial x}] e^{i\omega t}, y = d^- \qquad \text{(III-44)}$$

if the store airfoil executes a pitch oscillation about $x = x_0$ one has:

$$y = -\theta(x - x_0) e^{i\omega t} \qquad \text{(III-45)}$$

$$v(x, 0^+, t) = [-u_\infty \theta - i\omega\theta(x - x_0)] e^{i\omega t} \text{ at } y = 0^+ \qquad \text{(III-46)}$$

and similarly for the wing airfoil:

$$y = -\theta\left(x - B - x_0\right)e^{i(\omega t+\delta)} \qquad \text{(III-47)}$$

$$v\left(x,d^-,t\right) = \left[-\theta\left\{u_\infty \cos\delta - \omega.\sin\delta(x - B - x_0\right\} - i\theta\left\{u_\infty \sin\delta + \omega.\cos\delta(x - B - x_0)\right\}\right]e^{i\omega.t}$$

$$\text{(III-48)}$$

IV. ELEMENTARY THEORY

The case of supersonic leading edge locus will be examined assuming first that both airfoils are oscillating slowly in a supersonic flow. The airfoils execute vibrations of identical modes and amplitude but with different phase angle δ between them. It is obvious that the flow field is such that no disturbance can propagate upstream of the airfoil leading edges. Consequently, the perturbation potential can be written as:

$$\Phi(x,y,t) = \varphi(x,y,k)e^{ikt} \tag{IV-1}$$

where x, y are non dimensional coordinates and t is non dimensional time. The differential equation for the perturbation potential in a non-dimensional form then is:

$$\cot\alpha \cdot \varphi_{xx} - \varphi_{yy} + 2ikM^2\varphi_x - k^2M^2\varphi = 0 \tag{IV-2}$$

The linearized boundary conditions are:

$$\varphi_y(x,0) = v(x) \text{ on the lower(store airfoil) at } y=0 \tag{IV-3}$$

where $v(x)$ is the downwash amplitude.

$$\varphi_y(x,d) = -e^{i\delta}v(x - d\tan\beta) \text{ on the upper(wing) airfoil at } y=d \tag{IV-4}$$

where it is assumed that the two airfoils oscillate with a phase angle δ.

Assuming oscillation around x_0 $v(x)$ becomes:

$$v(x) = \theta[1 + ik(x - x_0)] \text{ and without loss of generality we can set } \theta=1 \tag{IV-5}$$

For small oscillations (low frequency) the potential φ can be expanded and neglecting the higher order terms of k one can write:

$$\varphi(x,y,k) = X(x,y) + k\Psi(x,y) \tag{IV-6}$$

Therefore the differential equation (II-2) becomes:

$$\cot^2\alpha \cdot X_{xx} - X_{yy} = 0 \tag{IV-7}$$

$$\cot\alpha \cdot \Psi_{yy} - \Psi_{yy} + 2iM^2 X_x = 0 \tag{IV-8}$$

A general solution of the above system of equations has been given by Sauer (1950). It reads:

$$X(x,y) = g(z) \tag{IV-9}$$

$$\Psi(x,y) = h(z) + iMyg(z)/\cos\alpha \quad \text{where } z = x - y\cot\alpha \tag{IV-10}$$

$$\Psi(x,y) = \bar{h}(z) + iMy\,\bar{g}(z)/\cos\alpha \quad \text{where } \bar{z} = x + y\cot\alpha \tag{IV-11}$$

$h(z)$ and $g(z)$ are arbitrary functions for positive arguments of z and zero for $z<0$, it is obvious that the expressions of the above functions should be consistent with the last assumption.

Using the two solutions for left and right running Mach waves in the supersonic flow field of slowly oscillating airfoils pressure coefficients can be obtained. In order to do so the flow field between the two airfoils has to be divided into several zones. The number of zones depends upon A. It is obvious that for $A>1$ where $A=d\cot\alpha$ there is one zone along each airfoil, for $0.5<A<1$ there are two, for $0.33<A<0.5$ there are three and so on. In the first zone I there is no interference from the airfoil wing (upper wing), which means that the lower airfoil (zone I only) does not sense the upper airfoil. In that case, the solution has the form:

$$\varphi(x,y,k) = g(z) + k\{h(z) - iMyg(z)/\cos\alpha\}$$

The boundary conditions that must be satisfied at $y=0$ are:

$$X_y = -\cot\alpha \cdot g'(z) = -1$$

$$\Psi_y = -\cot\alpha h'(z) - iMyg(z)/\cos\alpha = -i(x - x_0)$$

From the last two equations it is obvious that:

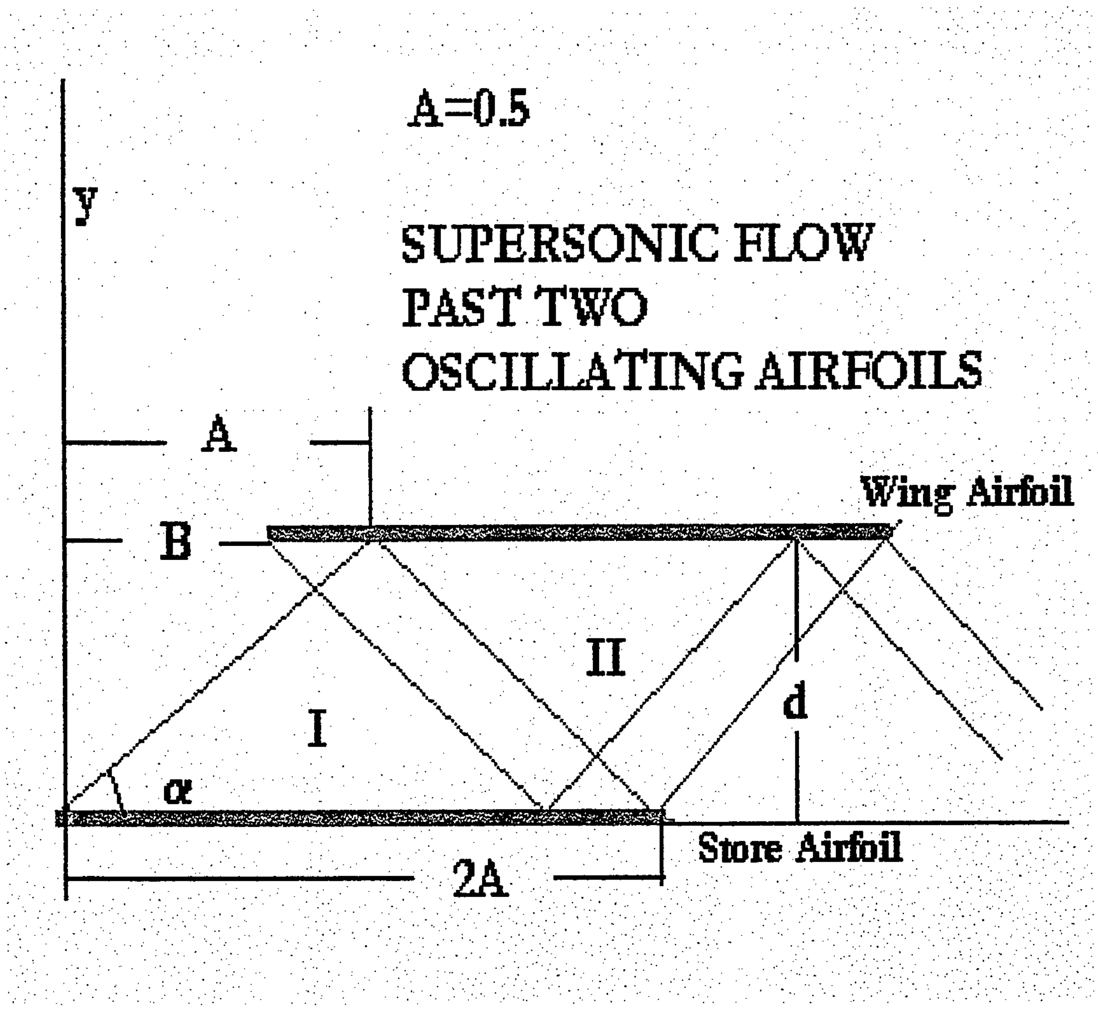
A=0.5
y
SUPERSONIC FLOW
PAST TWO
OSCILLATING AIRFOILS
A
B
Wing Airfoil
II
I
d
α
Store Airfoil
2A

Fig. 3

$g(z)=z\tan\alpha$

$h(z)=-iz\tan\alpha(x_0+z\tan^2\alpha/2)$

Since we are assuming a slowly oscillating airfoil, the solution to the PDE has the form:

$\varphi(x,y,k)=g(z)+k[h(z)-iMyg(z)/\cos\alpha]$

where $z=x-y\cot\alpha$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone I the following relation should be used:

$cp=-2[ik\varphi+\varphi_x]$

Substituting the known functions φ and φ_x

$\varphi=z\tan a+k\{-iz\tan\alpha(x_0+z\tan^2\alpha/2)-(iMyz\tan\alpha/\cos\alpha)\}$

$\varphi_x=d/dx[z\tan\alpha+k\{-iz\tan\alpha(x_0+z\tan^2\alpha/2)-(iMyz\tan\alpha/\cos\alpha)\}]=$

$=\tan\alpha-ik\tan\alpha[x_0+z\tan^2\alpha-(Myz/\cos\alpha)]=$

$=\tan\alpha-ik[\tan\alpha(x_0+z\tan^2\alpha)+(y/\sin\alpha\cos\alpha)(\sin\alpha/\cos\alpha)]=$

$=\tan\alpha-ik[\tan\alpha(x_0+z\tan^2\alpha)+(y/\cos^2\alpha)]=$

$=\tan\alpha-ik[\tan\alpha(x_0+(x-y\cot\alpha)\tan^2\alpha+(y/\cos^2\alpha)]=$

$=\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\tan\alpha+(y/\cos^2\alpha)]=$

$=\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\cot\alpha]=$

Therefore, the pressure distribution is given by the following relationship:

$$cp=-2\{\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\cot\alpha]+ik[z\tan\alpha+k\{-iz\tan\alpha(x_0+z\tan^2\alpha/2)-iMyz\tan\alpha/\cos\alpha\}]\}$$

Taking into consideration that higher order terms of k are neglected the pressure distribution is given by the following relationship:

$$cp=2\tan\alpha\{-1+ik[x_0+x(\tan^2\alpha-1)+2y\cot\alpha]\}$$

and therefore on the airfoil for y=0:

$$cp=2\tan\alpha\{-1+ik[x_0+x(\tan^2\alpha-1)]\}$$

In the zone I* the perturbation potential can also be written directly from Sauer's single airfoil solution:

$$\varphi(x,y,k)=g_1(z_1)+k[h_1(z_1)+iMyg(z_1)/\cos\alpha]$$

Where:

$$z_1=x-A-B+y\cot\alpha$$

The boundary conditions that must be satisfied at y=d are:

$$X_y=\cot\alpha\cdot g'(z_1)=-e^{i\delta}$$

$$\Psi_y=\cot\alpha\cdot h_1'(z_1)-iMg_1(z_1)/\cos\alpha+iMy\,g_1'(z_1)=-ie^{i\delta}(x-x_0-B):$$

After integration and algebraic manipulations:

$$g_1(z_1)=-z_1\tan\alpha\cdot e^{i\delta}$$

$$h_1(z_1)=iz_1\tan\alpha\cdot e^{i\delta}(x_0+z_1\tan^2\alpha/2+Md/\cos\alpha)$$

where again the condition has been imposed that the functions $g_1(z_1)$ and $h_1(z_1)$ are zero for negative and zero arguments.

Following the same procedure as in zone I, the perturbation potential φ will be calculated substituting the known functions $g_1(z_1)$ and $h_1(z_1)$:

$$\varphi = -z_1 \tan\alpha e^{i\delta} + ik\{z_1 \tan\alpha\, e^{i\delta}(x_0 + z_1 \tan^2\alpha/2 + Md/\cos\alpha) - Myz_1 \tan\alpha e^{i\delta}/\cos\alpha\}$$

$$\varphi_x = -\tan\alpha e^{i\delta} + ik\tan\alpha\, e^{i\delta}\{(x_0 + z_1 \tan^2\alpha + Md/\cos\alpha) - My/\cos\alpha\} =$$

$$= -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - A - B + y\cot\alpha)\tan^2\alpha + Md/\cos\alpha) - My/\cos\alpha\} =$$

$$= -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)\tan^2\alpha - A\tan^2\alpha + y\cot\alpha + d/\sin\alpha \cdot \cos\alpha)$$

$$-y/\sin\alpha\cos\alpha\} =$$

$$= -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)\tan^2\alpha + A - y\cot\alpha\}$$

Because $d = A/\cot\alpha$, $\quad \tan\alpha - 1/\cos\alpha \cdot \sin\alpha = \cot\alpha$

This gives the following result for the pressure distributions on the lower surface of the upper airfoil (wing airfoil) in zone I*:

$$cp = 2\tan\alpha \cdot e^{i\delta} - 2ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)(\tan^2\alpha - 1) + 2A - 2y\cot\alpha\} =$$

$$cp = 2\tan\alpha \cdot e^{i\delta}\{1 - ik\{(x_0 + (x - B)(\tan^2\alpha - 1) + 2A - 2y\cot\alpha\}$$

and therefore at $y = d$:

$$cp = 2\tan\alpha \cdot e^{i\delta}\{1 - ik\{(x_0 + (x - B)(\tan^2\alpha - 1)\}$$

Assuming that $0.5<A<1$ in addition to zones I, I* there are two more zones II, II*. In zone II the perturbation potential is due to the initial waves from zones I, I* plus the reflected wave at zone II.So the reflected waves must be determined by adding the functions $g_2(z_2)$ and $h_2(z_2)$. In the zone II Sauer's solution for a single airfoil applies and following the same procedure as previously:

$$\varphi(x,y,k) = g(z)+k[h(z)-iM(yg(z)/\cos\alpha\,]+g_1(z_1)+k[h_1(z_1)+iM(yg_1(z_1)/\cos\alpha]+$$

$$g_2(z_2)+k[h_2(z_2)-iMyg_2(z_2)/\cos\alpha]$$

The variable z has the following form:

$$z_2=x-2A+y\cot\alpha$$

This ensures that $z_2<0$ upstream of the leading edge of the airfoil.

The boundary conditions that must be satisfied at y=d are:

$$X_y=-\cot\alpha\cdot g'(z)+\cot\alpha\cdot g_1'(z_1)+\cot\alpha\cdot g_2'(z_2)=-e^{i\delta}$$

$$\Psi_y=-\cot\alpha\cdot h_1'(z)+\cot\alpha\cdot h_1'(z_1)+\cot\alpha\cdot h_2'(z_2)-iM[g(z)-g_1(z_1)-g_2(z_2)]/\cos\alpha+iMd\cot\alpha[g_1'(z_1)$$

$$+g_1'(z_1)+g_2'(z_2)]/\cos\alpha=-ie^{i\delta}(x-x_0-B)$$

After integration and algebraic manipulations similar to the previous calculations for zones I, I*:

$$g_2(z_2)=z\tan\alpha$$

$$h_2(z_2) =- iz_2\tan\alpha\,(x_0+ z_2/2\tan^2\alpha +2Md/\cos\alpha)$$

Therefore, the perturbation potential is given by the following relationship:

$$\phi_x=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha(x_0+z_2\tan^2\alpha+2Md/\cos\alpha)+ik\tan\alpha\cdot e^{i\delta}(x_0+z_1\tan^2\alpha+Md/\cos\alpha)$$

$$-ik\tan\alpha(x_0+z\tan^2\alpha)-ikMy[\tan\alpha-\tan\alpha+\tan\alpha\cdot e^{i\delta}]/\cos\alpha=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[(x_0+z_2\tan^2\alpha+2Md/\cos\alpha)-e^{i\delta}(x_0+z_1\tan^2\alpha+Md/\cos\alpha)-(x_0+z\tan^2\alpha$$

$$)+Mye^{i\delta}/\cos\alpha]=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[x_0+(x-2A+y\cot\alpha)\tan^2\alpha+2Md/\cos\alpha-e^{i\delta}(x_0+(x-A-B+y\cot\alpha)\tan^2\alpha$$

$$+Md/\cos\alpha)-(x_0+(x-y\cot\alpha)\tan^2\alpha)+Mye^{i\delta}/\cos\alpha]=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[x_0+(x-2A)\tan^2\alpha+2A\tan\alpha/\sin\alpha\cdot\cos\alpha-e^{i\delta}(x_0+(x-A-B+y\cot\alpha)\tan^2\alpha$$

$$+A\tan\alpha/\sin\alpha\cdot\cos\alpha)-(x_0+x\tan^2\alpha-ye^{i\delta}/\sin\alpha\cdot\cos\alpha)]=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[x_0+x\tan^2\alpha+2A-e^{i\delta}(x_0+(x-B+y\cot\alpha)\tan^2\alpha+A)+x_0+x\tan^2\alpha+ye^{i\delta}$$

$$/\sin\alpha\cdot\cos\alpha]=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[2x_0+2x\tan^2\alpha+2A-e^{i\delta}(x_0+(x-B+y\cot\alpha)\tan^2\alpha+A)+ye^{i\delta}/\sin\alpha\cdot\cos\alpha]=$$

$$=2\tan\alpha-\tan\alpha\cdot e^{i\delta}-ik\tan\alpha[(2-e^{i\delta})(x_0+x\tan^2\alpha+A)+e^{i\delta}(B\tan^2\alpha+y\cot\alpha)]=$$

$$=\tan\alpha\{2-e^{i\delta}-ik[(2-e^{i\delta})(x_0+x\tan^2\alpha+A)+e^{i\delta}(B\tan^2\alpha+y\cot\alpha)]\}$$

This gives the following result for the pressure distributions on the lower surface of the upper airfoil (wing airfoil) in zone II:

$$Cp=-\tan\alpha\{2-e^{i\delta}+ik[e^{i\delta}(x_0+(x-B)(\tan^2\alpha-1)+2A-2y\cot\alpha)-$$

$$2(x_0+x(\tan^2\alpha-1)+2A)\}$$

and for y=d we have:

$$Cp=-\tan\alpha\{2-e^{i\delta}+ik[e^{i\delta}(x_0+(x-B)(\tan^2\alpha-1))-2(x_0+x(\tan^2\alpha-1)+2A)\}$$

In zone II* the perturbation potential is due to the initial waves from zones I, I* plus the reflected wave at zone II, II*.So the reflected waves must be determined by adding the functions $g_3(z_3)$ and $h_3(z_3)$. Therefore, in the zone II* Sauer's solution for a single airfoil applies and following the same procedure as previously:

$$\varphi(x,y,k) = g(z)+k[h(z)-iM(yg(z)/\cos\alpha]+g_1(z_1)+k[h_1(z_1)+iM(yg_1(z_1)/\cos\alpha]+$$

$$g_3(z_3)+k[h_3(z_3)-iMyg_3(z_3)/\cos\alpha]$$

The variable z_3 has the following form:

$$z_3=x-A-B-y\cot\alpha$$

The boundary conditions that must be satisfied at y=0 are:

$$X_y=-\cot\alpha\cdot g'(z)+\cot\alpha\cdot g_1'(z_1)-\cot\alpha\cdot g_3'(z_3)=-1$$

$$\Psi_y=-\cot\alpha\cdot h'(z)+\cot\alpha\cdot h_1'(z_1)-\cot\alpha\cdot h_3'(z_3)-iM[g(z)-g_1(z_1)+g_3(z_3)]/\cos\alpha=-i(x-x_0)$$

After integration and algebraic manipulations similar to the previous calculations for zones I, I*:

$$g_3(z_3)=-z_3\tan\alpha\cdot e^{i\delta}$$

$$h_3(z_3)=iz_3\tan\alpha\,(x_0+z_3\tan^2\alpha/2+Md/\cos\alpha)e^{i\delta}$$

Therefore, the perturbation potential is given by the following relationship:

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\tan\alpha\{e^{i\delta}(x_0+z_1\tan^2\alpha+Md/\cos\alpha)+e^{i\delta}(x_0+z_3\tan^2\alpha+Md/\cos\alpha)-(x_0+z\tan^2\alpha)-My/\cos\alpha\}$$

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\tan\alpha\{e^{i\delta}[x_0+(x-A-B+y\cot\alpha)\tan^2\alpha+Md/\cos\alpha]e^{i\bar{\delta}}[x_0+(x-A-B-y\cot\alpha)\tan^2\alpha+Md/\cos\alpha]-(x_0+(x-y\cot\alpha)\tan^2\alpha)-My/\cos\alpha\}=$$

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\{e^{i\delta}2\tan\alpha[x_0+(x-A-B)\tan^2\alpha+Md/\cos\alpha]-\tan\alpha(x_0+(x-y\cot\alpha)\tan^2\alpha)-y/\cos^2\alpha\}$$

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\{e^{i\delta}2\tan\alpha[x_0+(x-B)\tan^2\alpha-A\tan^2\alpha+Ad\tan\alpha/\sin\alpha\cdot\cos\alpha]-\tan\alpha(x_0+(x-y\cot\alpha)\tan^2\alpha)-y/\cos^2\alpha\}=$$

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\{e^{i\delta}2\tan\alpha[x_0+(x-B)\tan^2\alpha+A]-\tan\alpha(x_0+(x-y\cot\alpha)\tan^2\alpha)-y/\cos^2\alpha\}=$$

$$\varphi_x=\tan\alpha-2\tan\alpha\cdot e^{i\delta}+ik\{e^{i\delta}2\tan\alpha[x_0+(x-B)\tan^2\alpha+A]-\tan\alpha(x_0+x\tan^2\alpha+y\cot\alpha)\}$$

Because: $y/\cos^2\alpha+y\cot\alpha\tan^3\alpha=-y$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone II* the following relation should be used:

$$cp=-2[ik\varphi+\varphi_x]$$

Substituting the known functions φ and φ_x

$$=2\tan\alpha-4\tan\alpha\cdot e^{i\delta}-2ik\{e^{i\delta}2\tan\alpha[x_0+(x-B)\tan^2\alpha+A]-\tan\alpha(x_0+x\tan^2\alpha+y\cot\alpha)\}$$

$-2ik\{z\tan\alpha-z_1\tan\alpha\cdot e^{i\delta}-z_3\tan\alpha\cdot e^{i\delta}\}$+higher order of k terms which will be neglected

Therefore:

$$Cp=2\tan\alpha-4\tan\alpha\cdot e^{i\delta}-2ik\{e^{i\delta}2\tan\alpha[x_0+(x-B)\tan^2\alpha+A]-\tan\alpha(x_0+x\tan^2\alpha+y\cot\alpha)\}$$

$$-2ik\{(x-y\cot\alpha)\tan\alpha-(x-A-B+y\cot\alpha)\tan\alpha\cdot e^{i\delta}-(x-A-B-y\cot\alpha)\tan\alpha\cdot e^{\overline{i\delta}}\}$$

After several relatively simple algebraic manipulations:

$$Cp=-2\tan\alpha[1-2e^{i\delta}-ik\{2e^{i\delta}[x_0+(x-B)(\tan^2\alpha-1)+2A]-(x_0+x(\tan^2\alpha-1)+2y\cot\alpha)\}]$$

Since the pressures have been calculated for the flow field between the airfoils a similar approach will be utilized to obtain the pressures on the bottom of the lower airfoil. In that case, the solution has the form:

$$\varphi(x,y,k) = g(z) + k\{h(z) + iMyg(z)/\cos\alpha\}$$

The boundary conditions that must be satisfied at y=0 are:

$$X_y = \cot\alpha\cdot g'(z) = -1$$

$$\Psi_y = \cot\alpha\cdot h'(z) + iMg(z)/\cos\alpha = -i(x-x_0) \quad \text{at } y=0$$

From the last two equations it is obvious that:

$$g(z) = z\tan\alpha$$

$$h(z) = -iz\tan\alpha(x + z\tan^2\alpha/2)$$

The solution again has the form:

$$\varphi(x,y,k) = g(z) + k[h(z) + iMyg(z)/\cos\alpha]$$

where $z = x + y\cot\alpha$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone I the following relation should be used:

$$cp = -2[ik\varphi + \varphi_x]$$

$\varphi=-z\tan\alpha + k\{-iz\tan\alpha(x+z\tan^2\alpha/2)+(iMyz\tan\alpha/\cos\alpha)\}$

$\varphi_x=d/dx[-z\tan\alpha+k\{-iz\tan\alpha(x+z\tan^2\alpha/2)+(iMyz\tan\alpha/\cos\alpha)\}]=$

$=-\tan\alpha- ik\tan\alpha[x+z\tan^2\alpha-(Myz/\cos\alpha)]=$

$=-\tan\alpha-ik[\tan\alpha(x_0+z\tan^2\alpha)-(y/\sin\alpha\cos\alpha)(\sin\alpha/\cos\alpha)]=$

$=\tan\alpha - ik[\tan\alpha(x_0+ z\tan^2\alpha)+(y/\cos^2\alpha)]=$

Therefore, the pressure distribution is given by the following relationship:

$cp=2\tan\alpha\{ 1-ik[\tan\alpha(x_0+ x (\tan^2\alpha-1)+2y\cot\alpha\}$

Using the results for zones I,I*,II,II* it is possible to calculate the pressure jump across the bottom airfoil (wing store airfoil) from the following relationship:

$\Delta cp(x,0)=cp(x,0^+)- cp(x,0^-)$

Two integration intervals, shown in Fig.4, can be distinguished in the lower airfoil taking into consideration that $0.5\leq A\leq 1$:

1) $0\leq x\leq A+B$
2) $A+B\leq x\leq 2A$

In the first integration interval:

$\Delta cp_I(x,0)=cp(x,0^+)- cp(x,0^-)=$

$=2\tan\alpha\{ 1-ik[\tan\alpha(x_0+x (\tan^2\alpha-1)+2y\cot\alpha\}$

$+2\tan\alpha\{ 1-ik[\tan\alpha(x_0+x (\tan^2\alpha-1)+2y\cot\alpha\}$ and $y=0$

$\Delta cp_I(x,0)=4\tan\alpha\{ 1-ik(x_0+ x(\tan^2\alpha-1)\}$

In the second integration interval:

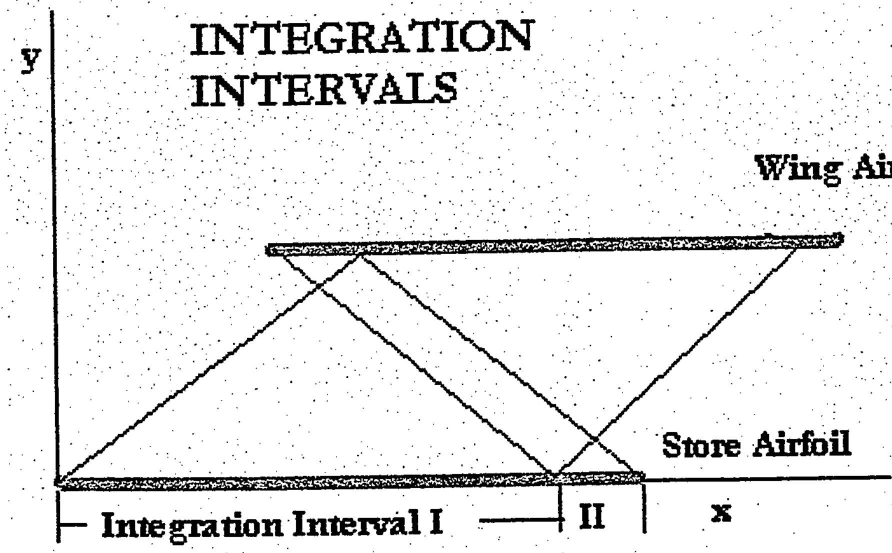

Fig.4

$\Delta cp_{II}(x,0) = 2\tan\alpha\{1-ik[\tan\alpha(x_0+ x(\tan^2\alpha-1)+2y\cot\alpha\}$

$+2\tan\alpha[1-2e^{i\delta}+ik\{2e^{i\delta}[x_0+(x-B)(\tan^2\alpha-1)+2A]-(x_0+x(\tan^2\alpha-1)+2y\cot\alpha)\}]$

$=+2\tan\alpha[1-2e^{i\delta}+ik\{2e^{i\delta}[x_0+(x-B)(\tan^2\alpha-1)+2A]-2(x_0+x(\tan^2\alpha-1))\}]$ since $y=0$

But $e^{i\delta}=\cos\delta+i\sin\delta$ and the last expression can be written:

$=+2\tan\alpha[1-2(\cos\delta+i\sin\delta)+ik\{2(\cos\delta+i\sin\delta)[x_0+(x-B)(\tan^2\alpha-1)+2A]-2(x_0+x(\tan^2\alpha-1))\}]$

This expression is a complex equation with real and imaginary part. In order to calculate the pitch-damping coefficient only the imaginary part of Δcp is needed. Therefore one has:

$\mathrm{Im}\Delta cp_I(x,0)=-4\tan\alpha(x_0+x(\tan^2\alpha-1))$

$$\mathrm{Im}\Delta cp_{II}(x,0)=+2\tan\alpha[-2\sin\delta/k+2\cos\delta[x_0+(x-B)(\tan^2\alpha-1)+2A]-2(x_0+x(\tan^2\alpha-1))]$$

The lift and moment coefficient can be calculated from the following relationship:

$$C_L={}_0\int^1\Delta cp\,dx$$

$$C_M={}_0\int^1\Delta cp(x-x_0)dx$$

By definition:

$$C_M=\theta[\,C_{M\theta}+ik\,C_{M\theta'}]$$

$$C_{M\theta'}=C_{M\theta'I}+C_{M\theta'II}=$$

$$(1/ik)_0\int^{A+B}\mathrm{Im}\Delta cp_I(x,0)\,(x-x_0)dx+{}_{A+B}\int^1\mathrm{Im}\Delta cp_{II}(x,0)(x-x_0)dx=$$

$${}_0\int^{A+B}-4\tan\alpha(x_0+x(\tan^2\alpha-1)(x-x_0)dx+{}_{A+B}\int2\tan\alpha[-2\sin\delta/k+2\cos\delta[x_0+(x-B)(\tan^2\alpha-1)+2A]-2(x_0+x(\tan^2\alpha-1))](x-x_0)dx$$

The last two integrals will be calculated separately. The first one is:

$$2\tan\alpha[-2x_0^2\,(A+B)+x_0(A+B)(A+B)-2(\tan^2\alpha-1)[x_0(A+B)^2/2-(A+B)^3/3)] \qquad \text{(IV-12)}$$

The second one can be calculated using Maple. The results are a complicate expression. Two calculations have been included for the convenience of the reader since in both integrals there is a common factor 2tanα. The result is:

$$2\tan\alpha\{2x_0+\tan^2\alpha\cdot B^3A^2B-\cos\delta\cdot\tan^2\alpha\cdot B^3/3+2\tan^2\alpha/3-\sin\delta\cdot A^2/k-\sin\delta\cdot B^2/k+\sin\delta/k+2\cos\delta/3$$
$$+2x_0\sin\delta\cdot A/k-2x_0^2\cos\delta\cdot B-2x_0^2\cos\delta\cdot A+4x_0\cos\delta\cdot A+2x_0\cos\delta\cdot B+2x_0\sin\delta\cdot B/k+2x_0^2A+2x_0^2B$$
$$+2\cos\delta\cdot\tan^2\alpha A^3/3+2\cos\delta\cdot AB^2-4x_0AB-2\sin\delta\cdot AB/k-2/3+4\cos\delta\cdot A^3/3+2B^3/3+2AB^2+2BA^2$$
$$-2\tan^2\alpha\cdot AB^2-2\tan^2\alpha\cdot AB^2-2\cos\delta\cdot\tan^2\alpha/3-2x_0\cos\delta-x_0\tan^2\alpha-\cos\delta\cdot B-2\cos\delta\cdot A-2x_0\sin\delta/k+2x_0^2$$
$$\sin\delta/k+2x_0^2\cos\delta-2x_0^2+x_0\tan^2\alpha\cdot A^2+x_0\cos\delta\cdot\tan^2\alpha+x_0\tan^2\alpha\cdot B^2+\cos\delta\cdot B\tan^2\alpha-2x_0\cos\delta\cdot B\tan^2\alpha+$$

$3\cos\delta\cdot A^2B-2\tan^2\alpha\cdot A^3/3-2\tan^2\alpha\cdot B^3/3+\cos\delta\cdot B^3/3+2A^3/3-2x_0B^2+2x_0\tan^2\alpha\cdot AB$

$-2x_0A^2-2x_0\cos\delta\cdot BA+x_0\cos\delta\cdot\tan^2\alpha\cdot B^2-x_0\cos\delta\cdot A^2-x_0\cos\delta\cdot\tan^2\alpha\cdot A^2\}$ (IV-13)

Therefore:

$$C_{M\theta'}=2\tan\alpha[-2x_0^2(A+B)+x_0(A+B)(A+B)-2(\tan^2\alpha-1)[x_0(A+B)^2/2-(A+B)^3/3)]+$$
$$2\tan\alpha\{2x_0+\tan^2\alpha\cdot B^3A^2B-\cos\delta\cdot\tan^2\alpha\cdot B^3/3+2\tan^2\alpha/3-\sin\delta\cdot A^2/k-\sin\delta\cdot B^2/k+\sin\delta/k+2\cos\delta/3$$
$$+2x_0\sin\delta\cdot A/k-2x_0^2\cos\delta\cdot B-2x_0^2\cos\delta\cdot A+4x_0\cos\delta\cdot A+2x_0\cos\delta\cdot B+2x_0\sin\delta\cdot B/k+2x_0^2A+2x_0^2B$$
$$+2\cos\delta\cdot\tan^2\alpha A^3/3+2\cos\delta\cdot AB^2-4x_0AB-2\sin\delta\cdot AB/k-2/3+4\cos\delta\cdot A^3/3+2B^3/3+2AB^2+2BA^2$$
$$-2\tan^2\alpha\cdot AB^2-2\tan^2\alpha\cdot AB^2-2\cos\delta\cdot\tan^2\alpha/3-2x_0\cos\delta-x_0\tan^2\alpha-\cos\delta\cdot B-2\cos\delta\cdot A-2x_0\sin\delta/k+2x_0^2$$
$$\sin\delta/k+2x_0^2\cos\delta-2x_0^2+x_0\tan^2\alpha\cdot A^2+x_0\cos\delta\cdot\tan^2\alpha+x_0\tan^2\alpha\cdot B^2+\cos\delta\cdot B\tan^2\alpha-2x_0\cos\delta\cdot B\tan^2\alpha+$$
$$3\cos\delta\cdot A^2B-2\tan^2\alpha\cdot A^3/3-2\tan^2\alpha\cdot B^3/3+\cos\delta\cdot B^3/3+2A^3/3-2x_0B^2+2x_0\tan^2\alpha\cdot AB$$
$$-2x_0A^2-2x_0\cos\delta\cdot BA+x_0\cos\delta\cdot\tan^2\alpha B^2-x_0\cos\delta\cdot A^2-x_0\cos\delta\cdot\tan^2\alpha\cdot A^2\} \qquad \text{(IV-14a)}$$

This is the major result of our investigation. It represents the pitch –damping coefficient of the lower airfoil in the presence of the upper airfoil. Note that both airfoils are oscillating in pitch. Also the two airfoil oscillation may lag by a phase angle δ.

If $B=0$ and $\delta=180^0$ the above result for the pitch damping coefficient is the same as that of an oscillating airfoil mounted close to a stationary airfoil or wall at a distance d/2(M. F. Platzer, H. G. Chalkey). As will be shown later the same result can be derived using the elementary theory. The pitch-damping coefficient for that case is given by the relation:

$$C_{M\theta'}=-2\tan\alpha\{x_0^2(4-2A)-x_0(3A^2-4A+2+(A^2-2)(\tan^2\alpha-1))+2A^3-2A-2(2-A^3)/3\ (\tan^2\alpha-1)]$$

(IV-14b)

Where $A=2h\cot\alpha$, and h is the airfoil distance.

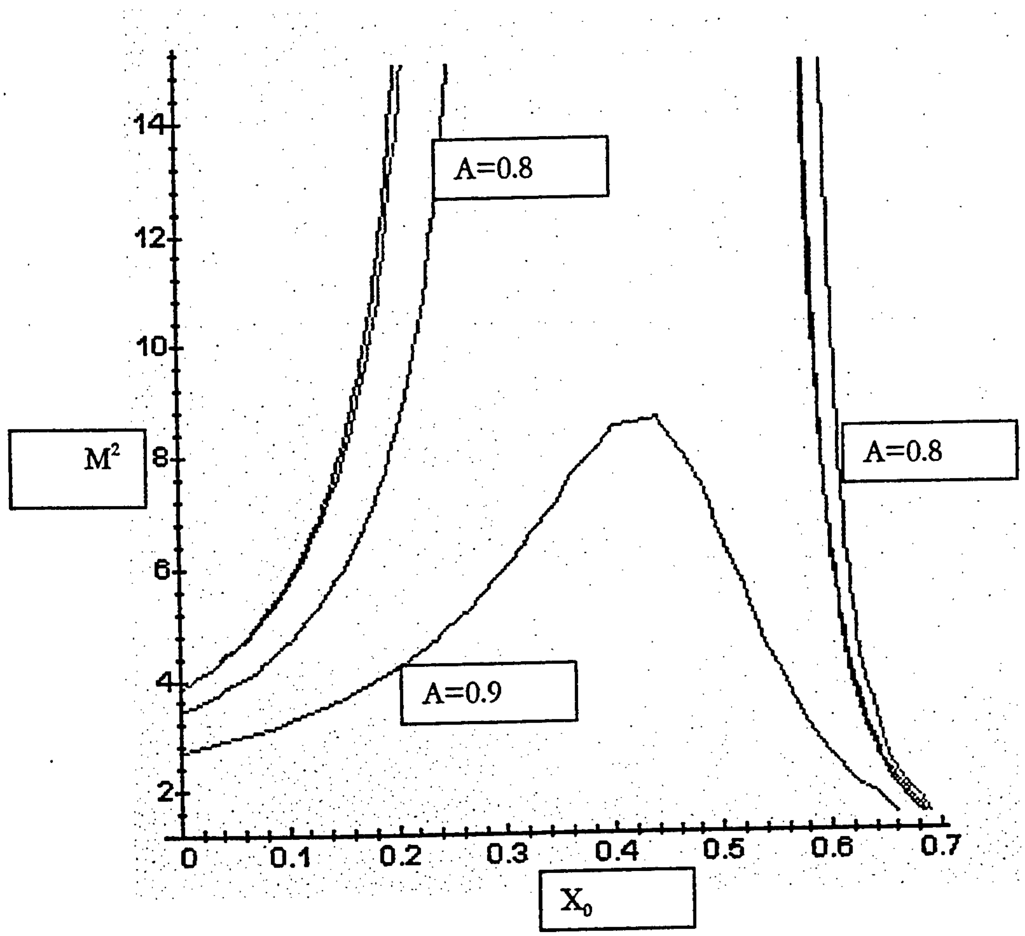

Fig. 5

For the special case of zero stagger B=0 and out-of-phase oscillation d=180 the general result equations (IV-12) and (IV-13) simplifies to (IV-14).

This agrees with the result previously obtained by Platzer and Chalkey (1972) if A=d.cota in replaced by A=2hcota in replaced by A=2hcota used by Platzer and Chalkey. Two unstaggered airfoils oscillating out-of-phase are equivalent to a single mounted at a distance h=d/2 from a solid wall of a non-moving airfoil.

It can be seen from Fig. 5 that the interference is highly destabilizing with increasing amounts of interference.

Note that A=0.8 implies a larger amount of interference than A=0.9, for example. At A=0.9 instability is encountered from low supersonic flow to a maximum of M=3, which at A=0.8 instability occurs up to much larger supersonic Mach numbers.

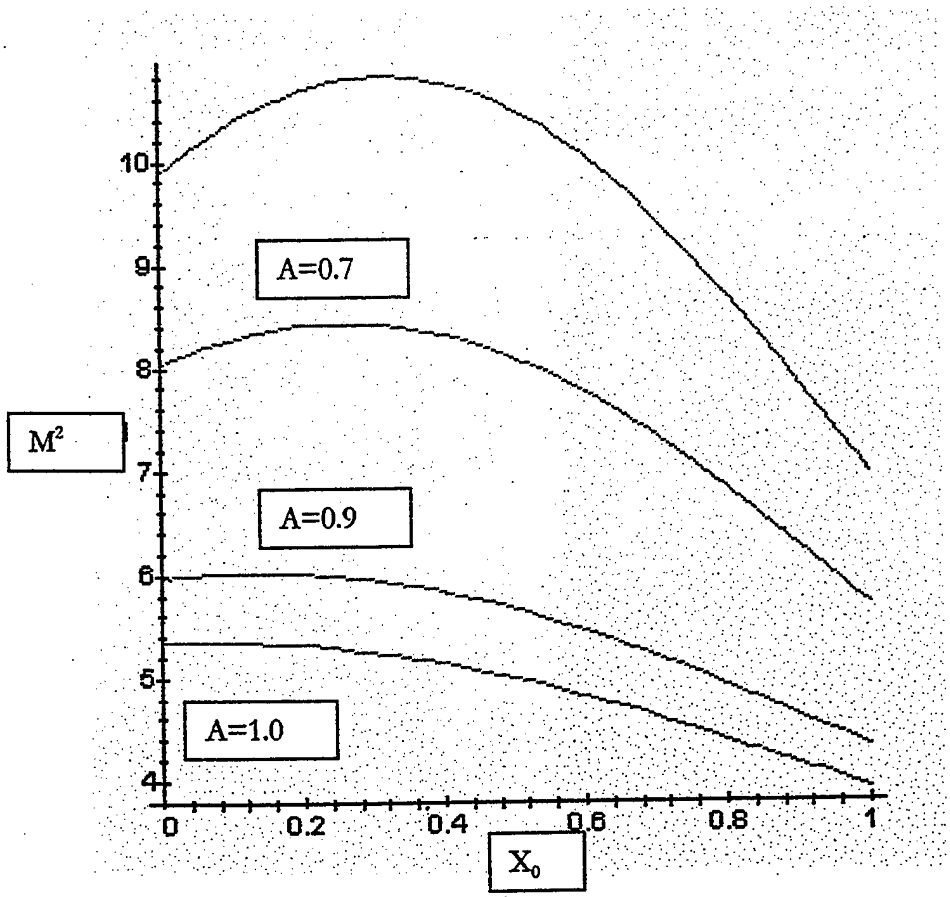

Fig. 6

Another interesting case occurs for in-phase oscillation δ=0 of two unstaggered airfoils. The instability boundaries are shown in Fig. 6. Again, it is seen that increasing amount of interference leads to increasing regions of instability.

Fig. 6 shows curves of zero pitch damping coefficient (with torsional stability boundaries) for an airfoil mounted close to another airfoil when B=0 but oscillating in phase.

Following the same procedure a special case of the above will be calculated. In that case the upper airfoil is assumed to be stationary while the lower airfoil (store airfoil) oscillates. The results will be compared with previous analyses (M.F. Platzer, 1971).

Starting again with Sauers solution for the slowly oscillating airfoil in unbounded supersonic flow it is shown how this solution can be extended to consider the upper airfoil (wing) interference. To further simplify the comparison it is assumed that:

$$\sqrt{M^2-1}\frac{2h}{c}\leq 1$$

Therefore, the moving airfoil is mounted at half distance from the stationary upper airfoil in compare to the previous analysis. The solution is given by the following set of equations:

$$X(x,y)=g(z)$$

$$\Psi(x,y)=h(z)+iMyg(z)/\cos\alpha \quad \text{where } z=x-y\cot\alpha$$

$$\Psi=h(z)+iMy\,g(z)/\cos\alpha \quad \text{where } z=x+y\cot\alpha$$

h(z) and g(z) are arbitrary functions for positive arguments of z and zero for z<0, it is obvious that the expressions of the above functions should be consistent with the last assumption.

Using the two solutions for left and right running Mach waves in the supersonic flow field of slowly oscillating airfoils pressure coefficients can be obtained. In order to do so the flow field between the two airfoils should be divided into several zones. The number of zones depends upon A. It is obvious that for A>1, there is one zone along the airfoil, for 0.5<A<1 there are two. So there are three regions. In the first zone I there is no interference from the airfoil wing (upper wing), which means that the lower airfoil (zone I only) does not sense the upper airfoil. In that case, the solution has the form:

$$\varphi(x,y,k)=g(z)+k\{h(z)-iMyg(z)/\cos\alpha\}$$

The boundary conditions that must be satisfied at y=0 are:

$X_y = -\cot\alpha \cdot g'(z) = -1$

$\Psi_y = -\cot\alpha \cdot h'(z) - iMyg(z)/\cos\alpha = -i(x - x_0)$

From the last two equations it is obvious that:

$g(z) = z\tan\alpha$

$h(z) = -iz\tan\alpha(x_0 + z\tan\alpha/2)$

Assuming a slowly oscillating airfoil, the solution to the PDE has the form:

$\varphi(x,y,k) = g(z) + k[h(z) - iMyg(z)/\cos\alpha]$

When $z = x - y\cot\alpha$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone I the following relation should be used:

$cp = -2[ik\varphi + \varphi_x]$

Substituting the known functions φ and φ_x

$\varphi = z\tan a + k\{-iz\tan\alpha(x_0 + z\tan^2\alpha/2) - (iMyz\tan a/\cos a)\}$

$\varphi_x = d/dx[z\tan\alpha + k\{-iz\tan\alpha(x_0 + z\tan^2\alpha/2) - (iMyz\tan\alpha/\cos\alpha)\}] =$

$= \tan\alpha - ik\tan\alpha[x_0 + z\tan^2\alpha - (Myz/\cos\alpha)] =$

$= \tan\alpha - ik[\tan\alpha(x_0 + z\tan^2\alpha) + (y/\sin\alpha \cdot \cos\alpha)(\sin\alpha/\cos\alpha)] =$

$= \tan\alpha - ik[\tan\alpha(x_0 + z\tan^2\alpha) + (y/\cos^2\alpha)] =$

$$=\tan\alpha-ik[\tan\alpha(x_0+(x-y\cot\alpha)\tan^2\alpha+(y/\cos^2\alpha)] =$$

$$=\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\tan\alpha+(y/\cos^2\alpha)] =$$

$$=\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\cot\alpha]$$

Therefore, the pressure distribution is given by the following relationship:

$$cp=-2\{\tan\alpha-ik[\tan\alpha(x_0+x\tan^2\alpha-y\cot\alpha]+ik[z\tan\alpha+k\{-iz\tan\alpha(x_0+z\tan^2\alpha/2)-iMyz\tan\alpha/\cos\alpha\}]\}$$

Taking into consideration that higher order terms of k are neglected the pressure distribution is given by the following relationship:

$$cp=2\tan\alpha\{-1+ik[x_0+x(\tan^2\alpha-1)+2y\cot\alpha]\}$$

So at $y=0$

$$cp=2\tan\alpha\{-1+ik[x_0+x(\tan^2\alpha-1)]\}$$

In the zone II:

$$\varphi(x,y,k)=g(z)+k[h(z)-iM(yg(z)/\cos\alpha]+g_1(z_1)+k[h_1(z_1)+iM(yg_1(z_1)/\cos\alpha]$$

where:

$$z_1=x-A+y\cot\alpha$$

In order to calculate the functions $g(z)$,$h(z)$ the boundary conditions will be applied.

The boundary conditions that must be satisfied at $y=h$ are:

$$X_y=-\cot\alpha\cdot g'(z)+\cot\alpha\cdot g_1'(z_1)=0$$

$$\Psi_y = -\cot\alpha \cdot h'(z) + \cot\alpha \cdot h_1'(z_1) - iM[g(z) - g_1(z_1)]/\cos\alpha + iMA\cot\alpha[g'(z) + \bar{g}_1'(z_1)]/2\cos\alpha = 0$$

After algebraic manipulation:

$$cp = -2\tan\alpha\{-1 + ik[x_0 + x(\tan^2\alpha - 1)] + 2A - 2y\cot\alpha\}$$

For the region III:

$$\varphi(x,y,k) = g(z) + k[h(z) - iM(yg(z)/\cos\alpha] + g_1(z_1) + k[h_1(z_1) + iM(yg_1(z_1/\cos\alpha] +$$

$$g_2(z_2) + k[h_2(z_2) - iMyg_2(z_2)/\cos\alpha]$$

where:

$$z_2 = x - A - y\cot\alpha$$

The boundary conditions that must be satisfied at $y=0$ are:

$$X_y = -\cot\alpha \cdot g'(z) + \cot\alpha \cdot g_1'(z_1) - \cot\alpha \cdot g_2'(z_2) = -1$$

$$\Psi_y = -\cot\alpha \cdot h'(z) + \cot\alpha \cdot h_1'(z_1) - \cot\alpha \cdot h_2'(z_2) - iM[g(z) - g_1(z_1) + g_2(z_2)]/\cos\alpha = -i(x - x_0)$$

After several manipulations:

$$cp = -2\tan\alpha\{3 + ik[-3x_0 + (3x - 2A)(\tan^2\alpha - 1)] - 2A/\cos^2\alpha\}$$

Using the results from the three zones, the pressure jump across the lower blade can be calculated from the following relationship

$$\Delta cp(x,0) = cp(x,0^+) - cp(x,0^-)$$

In order to calculate the $cp(x,0^-)$ previous results will be used (two airfoil oscillating case) for the lower surface of the store airfoil(store):

$cp=2\tan\alpha\{1+ik[x_0+x(\tan^2\alpha-1)]\}$

$So\Delta cp(x,0)=cp(x,0^+)-cp(x,0^-)$

Two integration intervals can be distinguished in the lower airfoil taking into consideration that $0.5\leq A\leq 1$:

1) $0\leq x\leq A$
2) $A\leq x\leq 2A$

In the first integration interval is:

$\Delta cp_I(x,0)=cp(x,0^+)-cp(x,0^-)=$

$=-2\tan\alpha\{1-ik[\tan\alpha(x_0+x(\tan^2\alpha-1)\}=$

$=-2\tan\alpha\{1-ik[\tan\alpha(x_0+x(\tan^2\alpha-1)\}$ and $y=0$

$\Delta cp_I(x,0)=4\tan\alpha\{1-ik(x_0+x(\tan^2\alpha-1)\}$

The latest expression is a complex number with real and imaginary part. In order to calculate the pitch-damping coefficient only the imaginary part of Δcp is needed. Therefore:

$Im\Delta cp_I(x,0)=4\tan\alpha\{-x_0+x(\tan^2\alpha-1)\}$

In the second integration interval:

$Im\Delta cp_{II}(x,0)=+2ik\tan\alpha[-4x_0+(4x-2A)(1-\tan^2\alpha)-2A/\cos^2\alpha]$

The lift and moment coefficient can be calculated from:

$C_L={}_0\int^1\Delta cp\,dx$

$C_M={}_0\int^1\Delta cp(x-x_0)dx$

By definition:

$C_M = \theta_0 [C_{M\theta} + ik\, C_{M\theta'}]$

$C_{M\theta'} = C_{M\theta'_I} + C_{M\theta'_{II}}$

For the purpose of the analysis, only the pitch-damping coefficient is required. Therefore, in order to facilitate the calculations only the necessary imaginary part of the C_M will be calculated.

$$C_{M\theta'} = C_{M\theta'_I} + C_{M\theta'_{II}} = (1/ik)\,{}_0\!\int^A \mathrm{Im}\Delta cp_I(x,0)\,(x-x_0)dx + {}_A\!\int^1 \mathrm{Im}\Delta cp_{II}(x,0)\,(x-x_0)dx =$$

$${}_0\!\int^A -4\tan\alpha(x_0 + x(\tan^2\alpha - 1)(x-x_0)dx + {}_A\!\int^1 2ik\tan\alpha[-4x_0 + (4x-2A)(1-\tan^2\alpha) - 2A/\cos^2\alpha](x-x_0)dx =$$

$$= -2\tan\alpha\{x_0^2(4-2A) - x_0(3A^2-4A+2+(A^2-2)(\tan^2\alpha-1)) + 2A^3 - 2A - 2(2-A^3)/3\ (\tan^2\alpha - 1)]$$

The above result is in agreement with the previous results derived by M. F. Platzer and H.G. Chalkey but the last one has been derived using the elementary theory.

Therefore the elementary theory that has been developed predicts the above result as a special case for B=0 and $\delta = 180^0$ and represents for that case the pitch damping coefficient of an oscillating airfoil mounted close to a stationary airfoil or wall at a distance h/2. It is also the same result of the analysis of supersonic flow past a slowly two-dimensional airfoil in a wind tunnel with porous wall as presented by M.F. Platzer, 1971.

The second general case that will be examined in the thesis is shown in fig. 2. In that case, the Mach cone is wide enough to include the wing airfoil.

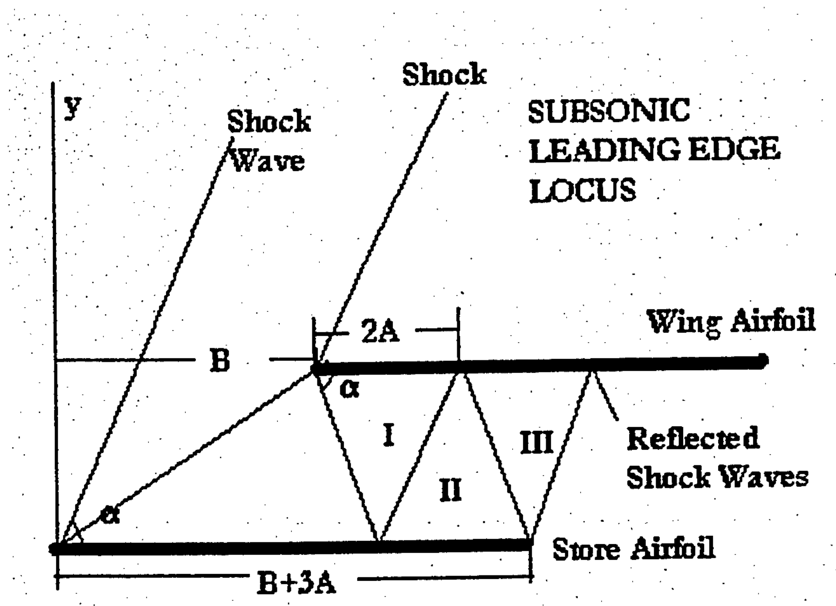

Fig.7

Using Sauer's solution as previously and dividing the flowfield in zones as depicted in Fig.7 the perturbation potential φ in zone I is given:

$$\varphi(x,y,k) = g(z) + k[h(z) + iMyg(z)/\cos\alpha]$$

Where:

$z = x - A - B + y\cot\alpha$ at $y=h$

The boundary conditions that must be satisfied at $y=h$ are:

$$X_y = \cot\alpha \cdot g'(z) = -e^{i\delta}$$

$\Psi_y = \cot\alpha \cdot h'(z) + iMg(z)/\cos\alpha + iMy\, g'(z)/\cos\alpha = -ie^{i\delta}(x - x_0 - B)$:

After integration and algebraic manipulations:

$g(z) = -z\tan\alpha \cdot e^{i\delta}$

$h(z) = iz\tan\alpha \cdot e^{i\delta}(x_0 + z\tan^2\alpha/2 + Mh/\cos\alpha)$

Where again the condition has been imposed that the functions g(z) and h(z) are zero for negative and zero arguments.

$\phi_x = -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + z_1\tan^2\alpha + Mh/\cos\alpha) - My/\cos\alpha\}$

$= -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - A - B + y\cot\alpha)\tan^2\alpha + Mh/\cos\alpha) - My/\cos\alpha\}$

$= \tan\alpha \cdot e^{i\delta} - ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)\tan^2\alpha - A\tan^2\alpha + y\cot\alpha + h/\sin\alpha \cdot \cos\alpha)$

$y/\sin\alpha \cdot \cos\alpha\} =$

$= -\tan\alpha \cdot e^{i\delta} + ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)\tan^2\alpha + A - y\cot\alpha\}$

Because $h = A/\cot\alpha$, $\tan\alpha - 1/\cos\alpha \cdot \sin\alpha = \cot\alpha$

This gives the following result for the pressure distributions on the lower surface of the upper airfoil (wing airfoil) in zone I:

$cp = -2[ik\varphi + \varphi_x]$

$cp = 2\tan\alpha \cdot e^{i\delta} - 2ik\tan\alpha \cdot e^{i\delta}\{(x_0 + (x - B)(\tan^2\alpha - 1) + 2A - 2y\cot\alpha\} =$

$cp = 2\tan\alpha \cdot e^{i\delta}\{1 - ik\{(x_0 + (x - B)(\tan^2\alpha - 1) + 2A - 2y\cot\alpha\}$

$$cp = 2\tan\alpha \cdot e^{i\delta}\{1 - ik\{(x_0 + (x - B)(\tan^2\alpha - 1)\}$$

In order to calculate the pressure on the upper surface of the wing airfoil the same approach will be followed and the result is:

$$cp = 2\tan\alpha \cdot e^{i\delta}\{-1 + ik\{(x_0 + (x - B)(\tan^2\alpha - 1)\}$$

To calculate the Δcp of the upper airfoil (wing airfoil) in the interval of zone I we have to calculate:

$$\Delta cp(x,h) = cp(x,h^-) - cp(x,h^+)$$

Therefore:

$$\Delta cp = 4\tan\alpha \cdot e^{i\delta}\{1 - ik\{(x_0 + (x - B)(\tan^2\alpha - 1)\}$$

Following the same arguments as previously for the zone II:

$$z_1 = x - A - y\cot\alpha - B$$

$$g_1(z_1) = z_1\tan\alpha \cdot (1 - e^{i\delta})$$

The second boundary condition gives:

$$-\cot\alpha \cdot h_1'(z) + \cot\alpha \cdot h'(z) - iM[-g(z) + g_1(z_1)]/\cos\alpha = -i(x - x_0 - B - A)$$

$$-\cot\alpha \cdot h_1'(z) + i\cot\alpha \cdot \tan\alpha e^{i\delta}(x_0 + z\tan^2\alpha + Mh/\cos\alpha) - iMz_1\tan\alpha/\cos\alpha + i(x - x_0 - B - A) = 0$$

Using the following trigonometric identity:

$z\tan^2\alpha - z/\cos^2\alpha = -z$ we have:

$h_1(z_1) = iz_1\tan\alpha\{(x_0 + z_1\tan^2\alpha/2)(-1 + e^{i\delta}) + Mh\, e^{i\delta}/\cos\alpha\}.$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone II the following relation should be used:

$$cp = -2[ik\varphi + \varphi_x]$$

Substituting the known functions φ and φ_x

$$\varphi = z_1\tan\alpha\cdot(1-e^{i\delta}) - z\tan\alpha\cdot e^{i\delta} + k\{iz_1\tan\alpha\{x_0 + z_1\tan^2\alpha/2)(-1+e^{i\delta}) + Mhe^{i\delta}/\cos\alpha\} + iz\tan\alpha\cdot e^{i\delta}(x_0 + z\tan^2\alpha/2 + Mh/\cos\alpha)\}$$

$$\varphi_x = d/dx[z_1\tan\alpha\cdot(1-e^{i\delta}) - z\tan\alpha\cdot e^{i\delta} + k\{iz_1\tan\alpha\{x_0 + z_1\tan^2\alpha/2)(-1+e^{i\delta}) + Mhe^{i\delta}/\cos\alpha\} + iz\tan\alpha\, e^{i\delta}(x_0 + z\tan^2\alpha/2 + Mh/\cos\alpha).\}] =$$

$$= \tan\alpha\cdot(1-e^{i\delta}) - \tan\alpha\cdot e^{i\delta} + k\{i\tan\alpha\{(x_0 + z_1\tan^2\alpha)(-1+e^{i\delta}) + Mhe^{i\delta}/\cos\alpha\} + i\tan\alpha\cdot e^{i\delta}(x_0 + z\tan^2\alpha + Mh/\cos\alpha)\}$$

$$= \tan\alpha\cdot(1-2e^{i\delta}) + ki\tan\alpha[\{(x_0 + z_1\tan^2\alpha)(-1+e^{i\delta}) + Mhe^{i\delta}/\cos\alpha\} + e^{i\delta}(x_0 + z\tan^2\alpha + Mh/\cos\alpha)]$$

Taking into consideration that y=0 we have:

$$= \tan\alpha\{(1-2e^{i\delta}) + ki[(x_0 + z_1\tan^2\alpha)(-1+2e^{i\delta}) + 2Mhe^{i\delta}/\cos\alpha]\}$$

$$cp = -2[ik\varphi + \varphi_x] = -2[\tan\alpha\cdot(1-2e^{i\delta}) + ki\tan\alpha\{(x_0 + z_1\tan^2\alpha - z_1 1)(-1+2e^{i\delta}) + 2Mhe^{i\delta}/\cos\alpha\}]$$

The last relation gives the cp on the upper surface of the lower airfoil (store) in the zone II.In the zone III a similar approach will be used to calculate the $h_2(z_2)$ and $g_2(z_2)$. The variable z_2, taking into consideration the fig. 2, is given by the following relationship:

$z_2 = x - 2A + y\cot\alpha - B$

The first boundary condition that must be satisfied at $y=h$ is:

$$g_2(z_2) = z_2 \tan\alpha \cdot (1-e^{i\delta})$$

The second boundary condition gives:

$$-\cot\alpha \cdot h_1'(z_1)+\cot\alpha \cdot h'(z)+\cot\alpha \cdot h_2'(z_2)+iM[g(z)-g_1(z_1)+g_2(z_2)]/\cos\alpha+iMh\cot\alpha[g'(z)+g_1'(z_1)$$
$$+g_2'(z_2)]/\cos\alpha=-i(x-x_0-B-2A)\, e^{i\delta}$$

$$\cot\alpha \cdot h_2'(z_2)+i\cot\alpha \cdot \tan\alpha(x_0+z_2\tan^2\alpha)-e^{i\delta}Mz_2\tan\alpha/\cos\alpha+iMh(2-3e^{i\delta})/\cos\alpha+i(x-x_0-B-2A)e^{i\delta}$$
$$=0$$

$$\cot\alpha \cdot h_2'(z_2)=-i(x_0+z_2\tan^2\alpha)+e^{i\delta}z_2/\cos^2\alpha- iMh(2-3e^{i\delta})/\cos\alpha-i(z_2-x_0)\, e^{i\delta}$$

$$h_2(z_2)=-iz_2\tan\alpha\ (x_0+z_2\tan^2\alpha/2)(1-e^{i\delta})- iMh\tan\alpha\ \ z_2\ (2-3e^{i\delta})/\cos\alpha$$

In a similar manner, as previously, the cp can be calculated:

$$\varphi=z_1\tan\alpha \cdot (1-e^{i\delta})-z\tan\alpha \cdot e^{i\delta}+z_2\tan\alpha \cdot (1-e^{i\delta})+k\{iz_1\tan\alpha\{x_0+z_1\tan^2\alpha/2)(-1+e^{i\delta})+Mhe^{i\delta}/\cos\alpha\}$$
$$+iz\tan\alpha \cdot e^{i\delta}(x_0+z\tan^2\alpha/2+Mh/\cos\alpha)+i\tan\alpha(x_0+z_2\tan^2\alpha/2)(1-e^{i\delta})-iMh\tan\alpha z_2(2-3e^{i\delta})/\cos\alpha$$
$$-iMy[g(z)-g_1(z_1)+g_2(z_2)]/\cos\alpha\}$$

$$\varphi_x=\tan\alpha \cdot (2-3e^{i\delta})+ki\tan\alpha[(x_0+z_1\tan^2\alpha)(-1+e^{i\delta})+Mhe^{i\delta}/\cos\alpha\}+e^{i\delta}(x_0+z\tan^2\alpha+Mh/\cos\alpha)+$$
$$(x_0+ z_2\tan^2\alpha)(-e^{i\delta}+1)-Mh(2-4e^{i\delta})/\cos\alpha]$$

$$\varphi_x=\tan\alpha \cdot (2-3e^{i\delta})+ki\tan\alpha[(x_0+z_1\tan^2\alpha)\, e^{i\delta}-Mh(2-6e^{i\delta})/\cos\alpha]$$

In order to calculate the pressure distribution on the upper surface of the lower airfoil in the zone II the following relation should be used:

$cp=-2[ik\varphi+\varphi_x]$

Substituting the known functions φ and φ_x

$$cp=-2ik\tan\alpha\cdot[(2-3e^{i\delta})+z_2\,(2-3e^{i\delta})+(x_0+z_2\tan^2\alpha)e^{i\delta}-Mh(2-6e^{i\delta})/\cos\alpha]$$

$$=-2ik\tan\alpha\cdot[(2-3e^{i\delta})+2z_2(1-e^{i\delta})+(x_0+(z_2-1)\tan^2\alpha)e^{i\delta}-Mh(2-6e^{i\delta})/\cos\alpha]$$

V. METHOD OF CHARACTERISTICS

In the method of characteristics, it is desirable to define the coordinate system in such a way that all-possible discontinuities could occur across. Along this coordinate system, the equations of motion of the flow field can then be treated as ordinary differential equations that are solvable by classical or numerical techniques.

To obtain the equations in this coordinate system (the characteristic directions) in the (x,y) plane, the equations of motion are written in terms of the arbitrary intersecting coordinates:

$$\xi = \xi(x, y) \tag{V-1}$$

And

$$\eta = \eta(x, y) \tag{V-2}$$

If the first derivatives of the dependent variables, $\overline{u}, v, p$ with respect to ξ are made indeterminate across lines of η = constant, and the first derivatives of the dependent variables with respect to η are made indeterminate across lines of ξ= constant, then any possible discontinuities in the first derivatives will occur across these lines. These lines are then the characteristics and their equations are obtained in the evaluation of the indeterminacies. Consider first, two-dimensional, steady flow. The governing equations of motion are the continuity equation:

$$\frac{\partial(\rho.\overline{u})}{\partial x} + \frac{\partial(\rho.v)}{\partial y} = 0 \tag{V-3}$$

The Euler equation becomes:

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+v\frac{\partial \bar{u}}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \qquad \text{(V-4)}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}=0 \qquad \text{(V-5)}$$

In addition, the energy equation becomes:

$$\bar{u}\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}=0 \qquad \text{(V-6)}$$

Along a streamline:

$$\bar{u}\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}=\bar{c}^{2}[\bar{u}\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}] \qquad \text{(V-7)}$$

Substituting the above into the continuity equation one obtains:

$$\rho\left[\frac{\partial \bar{u}}{\partial x}+\frac{\partial v}{\partial y}\right]=-\frac{1}{\bar{c}^{2}}\left[\bar{u}\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right] \qquad \text{(V-8)}$$

In terms of the new coordinates:

$$\rho\xi_x\bar{u}_\xi+\rho\xi_y v_\xi+\frac{1}{\bar{c}^{2}}\left[\bar{u}\xi_x+v\xi_y\right]p_\xi=-\rho\eta_x\bar{u}_\eta-\rho\eta_y v_\eta-\frac{1}{\bar{c}^{2}}\left[\bar{u}\eta_x+v\eta_y\right]p_\eta$$

(V-9)

In similar fashion the Euler equations become:

$$\rho\left[\bar{u}\xi_x+v\xi_y\right]\bar{u}_\xi+\xi_x p_\xi=-\rho\left[\bar{u}\eta_x+v\eta_y\right]\bar{u}_\eta-p_\eta\eta_x \qquad \text{(V-10)}$$

$$\rho\left[\bar{u}\xi_x+v\xi_y\right]v_\xi+\xi_x p_\xi=-\rho\left[\bar{u}\eta_x+v\eta_y\right]v_\eta-p_\eta\eta_x \qquad \text{(V-11)}$$

The last three equations form a system of three equations in $\bar{u}_\xi, v_\xi, p_\xi$. Solving for p_ξ by Cramer's rule and taking into consideration that p_ξ is indeterminate across η both determinants in numerator and denominator should vanish. Therefore, from the denominator determinant we have:

$$[\bar{u}\xi_x + v\xi_y]\cdot[(\bar{u}^2 - \bar{c}^2)\xi_x^{\ 2} + 2v\bar{u}\xi_x\xi_y + (v^2 - \bar{c}^2)\xi_y^{\ 2}] = 0 \qquad \text{(V-12)}$$

The solution to the above equation gives the equations of all three characteristics in the physical (x,y) plane:

$$\frac{dy}{dx} = \frac{v}{u} = \tan\zeta \qquad \text{(V-13)}$$

$$\frac{dy}{dx} = \tan(\zeta \pm \alpha) \qquad \text{(V-14)}$$

Where ζ is the angle the streamlines makes with x-axis and α is the Mach angle. The first equation describes a streamline while the last one describes left and right running Mach lines. The compatibility equation for $\bar{u}, v, p$ along ξ and η characteristics can be obtained by setting the numerator of p_ξ equal to zero:

$$v\frac{\partial \bar{u}}{\partial \eta} - u\frac{\partial v}{\partial \eta} \mp \frac{1}{\rho}\cot\alpha\frac{\partial p}{\partial \eta} = 0 \qquad \text{(V-15)}$$

The above relation must be satisfied along characteristics (ξ,η).

According to Teipel (1962) the unsteady flow over a flat plate can be treated in a similar fashion. In order to facilitate the calculations the continuity and Euler equations can be rewritten using the local sonic velocity:

$$\bar{c}^2 = \gamma\frac{p}{\rho} \qquad \text{(V-16)}$$

So canceling higher order terms the continuity equation becomes:

$$\frac{2}{\gamma-1}\frac{\partial \bar{c}}{\partial t}+\frac{2}{\gamma-1}u_\infty\frac{\partial \bar{c}}{\partial x}+c_\infty\frac{\partial \bar{u}}{\partial x}+c_\infty\frac{\partial v}{\partial x}=0 \qquad \text{(V-17)}$$

In a similar, fashion the Euler equations become:

$$\frac{\partial \bar{u}}{\partial t}+\frac{2}{\gamma-1}c_\infty\frac{\partial \bar{c}}{\partial x}+u_\infty\frac{\partial \bar{u}}{\partial x}=0 \qquad \text{(V-18)}$$

$$\frac{\partial v}{\partial t}+\frac{2}{\gamma-1}c_\infty\frac{\partial \bar{c}}{\partial x}+u_\infty\frac{\partial v}{\partial y}=0 \qquad \text{(V-19)}$$

A system of three differential equations has been formed for three unknowns $\bar{u}, v, \bar{c}$

To simplify the calculations with the assumption of harmonic oscillation we introduce the amplitude functions:

$$U(x,y)e^{i\omega t}=\frac{\bar{u}-u_\infty}{u_\infty} \qquad \text{(V-20)}$$

$$V(x,y)e^{i\omega t}=\frac{1}{\sqrt{M^2-1}}\frac{v}{u_\infty} \qquad \text{(V-21)}$$

$$C(x,y)e^{i\omega t}=\frac{2}{\sqrt{M^2-1}}\frac{1}{M^2}\frac{\bar{c}-c_\infty}{c_\infty} \qquad \text{(V-22)}$$

Therefore, the continuity and Euler equations become:

$$\frac{\partial U}{\partial x}+\sqrt{M^2-1}\frac{\partial V}{\partial y}+M^2\frac{\partial C}{\partial x}+ikM^2C=0 \qquad \text{(V-23)}$$

$$\frac{\partial U}{\partial x}+\frac{\partial C}{\partial x}+ikU=0 \qquad \text{(V-24)}$$

$$\frac{\partial V}{\partial x}+\frac{1}{\sqrt{M^2-1}}\frac{\partial C}{\partial y}+ikV=0 \tag{V-25}$$

With the above variables the compatibility relations for the above system of equations are:

$$\left(\frac{\partial V}{\partial x}\right)_\xi+\left(\frac{\partial C}{\partial x}\right)_\xi+ik\left[V+\frac{1}{M^2-1}\left(M^2C-U\right)\right]=0 \tag{V-26}$$

$$\left(\frac{\partial V}{\partial x}\right)_\eta-\left(\frac{\partial C}{\partial x}\right)_\eta+ik\left[V-\frac{1}{M^2-1}\left(M^2C-U\right)\right]=0 \tag{V-27}$$

$$\left(\frac{\partial U}{\partial x}\right)_{str}+\left(\frac{\partial C}{\partial x}\right)_{str}+ikU=0 \tag{V-28}$$

Taking into consideration the irrotationality:

$$\frac{\partial \bar{u}}{\partial y}-\frac{\partial v}{\partial x}=0 \tag{V-29}$$

The non-dimensional system of equations can be written as:

$$\frac{\partial U}{\partial x}+\sqrt{M^2-1}\frac{\partial V}{\partial y}+M^2\frac{\partial C}{\partial x}+ikM^2C=0 \tag{V-30}$$

$$\frac{\partial U}{\partial x}+\frac{\partial C}{\partial x}+ikU=0 \tag{V-31}$$

$$\frac{\partial U}{\partial y}-\sqrt{M^2-1}\frac{\partial V}{\partial x}=0 \tag{V-32}$$

Transforming the last system of equations to the new coordinate system (ξ,η):

$$\left(\frac{\partial U}{\partial x}\right)_\xi-\left(\frac{\partial V}{\partial x}\right)_\xi+ik\frac{M^2}{M^2-1}\left(U-C\right)=0 \tag{V-33}$$

$$\left(\frac{\partial U}{\partial x}\right)_\eta + \left(\frac{\partial V}{\partial x}\right)_\eta + ik\frac{M^2}{M^2-1}(U-C)=0 \qquad \text{(V-34)}$$

$$\left(\frac{\partial U}{\partial x}\right)_{str} + \left(\frac{\partial C}{\partial x}\right)_{str} + ikU = 0 \qquad \text{(V-35)}$$

The last system of equations can be solved using finite differences. The values of U,V and C on the upper and lower surfaces of the airfoils can be calculated using the results of the Elementary Theory (Chapter III). The following relations apply:

$$U = \varphi_x, V = \tan\alpha\varphi_y, C = -[\varphi_x + ik\varphi] \qquad \text{(V-36)}$$

Using the results of chapter III the following results apply to zone I:

$$U_I = \tan\alpha\left[1 - ik(x_0 + x\tan^2\alpha + y\cot\alpha)\right] \qquad \text{(V-37)}$$

$$V_I = \tan\alpha\left[-1 + ik(x_0 - x + y(\cot\alpha + \frac{M}{\cos\alpha}))\right] \qquad \text{(V-38)}$$

$$C_I = \tan\alpha\left[-1 + ik(x_0 + x(\tan^2\alpha - 1) + 2y\cot\alpha)\right] \qquad \text{(V-39)}$$

The last three equations represent the values of the parameters on the upper surface of the lower airfoil in the interval of the zone I. The last equation gives the pressure distribution and it is half the value of the actual pressure distribution coefficient. Following a similar approach the following results apply in the zone II* for the pressure coefficient:

$$\begin{aligned} C = 2\{&2\tan\alpha e^{i\delta} - \tan\alpha + ik\tan\alpha[x_0 + x(\tan^2\alpha - 1)] - \\ &2ike^{i\delta}\tan\alpha[x_0 + (\tan^2\alpha - 1)(x - B) + 2d\cot\alpha]\} \end{aligned} \qquad \text{(V-40)}$$

VI. FLUTTER ANALYSIS

The lift and moment are given from the following relations:

$$L = \frac{1}{2}\rho U^2 \int_0^1 C_p dx \qquad \text{(VI-1)}$$

$$M = \frac{1}{2}\rho U^2 \int_0^1 C_p (x - x_0) dx \qquad \text{(VI-2)}$$

And

$$Cp = -\frac{2}{U^2}\left(\frac{\partial \varphi}{\partial t} + U\frac{\partial \varphi}{\partial x}\right) \qquad \text{(VI-3)}$$

While

$$c_L = \int_0^1 \Delta c_p dx \qquad \text{(VI-4)}$$

$$c_M = \int_0^1 (x - x_0)\Delta c_p dx \qquad \text{(VI-5)}$$

And defining:

$$c_m = \theta_0 [c_{m\theta} + ikc_{m\theta'}] \qquad \text{(VI-6)}$$

The last relations have been used in the elementary theory as shown in chapter III, and are a result of the following analysis. To compute the non dimensional lift and moment acting on the store airfoil the non-dimensional pressure coefficient P as defined in problem formulation last equation will be utilized and consequently the lift and moment can be written as follows:

$$L = \int_0^1 \left\{ P(x,0^+) - P(x,0^-) \right\} dx \qquad \text{(VI-7)}$$

$$M = \int_0^1 \left\{ P(x,0^+) - P(x,0^-) \right\} (x - x_0) dx \qquad \text{(VI-8)}$$

And:

$$P(x,y) = \gamma M^2 C(x,y) \qquad \text{(VI-9)}$$

Where:

$$P(x,y)e^{i\omega t} = \frac{p - p_\infty}{p_\infty}, \qquad \text{(VI-10)}$$

Although in the thesis only pitching, oscillation is considered the more general case include plunging oscillation also. In that case, Garrick and Rubinow's (1946) method of expressing lift and pitching moment is used:

$$L = -\frac{1}{2}\rho_\infty \bar{c} u_\infty{}^2 k^2 e^{i\omega t}\left[\frac{2h_0}{\bar{c}}(L_1 + iL_2) + \theta_0(L_3 + iL_4)\right] \qquad \text{(VI-11)}$$

$$M = -\frac{1}{4}\rho_\infty \bar{c}^2 u_\infty{}^2 k^2 e^{i\omega t}\left[\frac{2h_0}{\bar{c}}(M_1 + iM_2) + \theta_0(M_3 + iM_4)\right] \qquad \text{(VI-12)}$$

Where:

$$k = \frac{\omega \bar{c}}{u_\infty} \text{ and } \bar{c} = 2b$$

For a single degree of freedom as in that case the flutter analysis is much easier, and the equation of motion is:

$$I_\theta \ddot{\theta} + (1 + ig)C_\theta \theta = M_\theta \qquad \text{(VI-13)}$$

The sum of the moments of inertia about the elastic axis is:

$$M_I = -I_\theta \ddot{\theta} \qquad \text{(VI-14)}$$

In addition, the elastic restoring moment is:

$$M_R = -(1+ig)C_\theta \theta \qquad \text{(VI-15)}$$

Moreover, the aerodynamic pitching moment about the elastic axis is:

$$M_\theta = -\frac{1}{4}\rho_\infty \bar{c} u_\infty{}^2 k^2 e^{i\omega t}\theta_0 (M_3 + iM_4) \qquad \text{(VI-16)}$$

Since:

$$\theta = \theta_0 e^{i\omega t} \qquad \text{(VI-18)}$$

$$\ddot{\theta} = -\omega^2 \theta_0 e^{i\omega t} \qquad \text{(VI-19)}$$

So the equation of motion becomes:

$$e^{i\omega t}[-I_\theta \omega^2 \theta_0 + (1+ig)C_\theta \theta_0 = -\frac{1}{4}\rho_\infty \bar{c}^2 u_\infty{}^2 k^2 e^{i\omega t}\theta_0 (M_3 + iM_4) \qquad \text{(VI-20)}$$

Separating the equation in real and imaginary parts gives:

$$-I_\theta \omega^2 + C_\theta + \frac{1}{4}\rho_\infty \hat{c}^4 \omega^2 M_3 = 0 \qquad \text{(VI-21)}$$

$$gC_\theta + \frac{1}{4}\rho_\infty \hat{c}^4 \omega^2 M_4 = 0 \qquad \text{(VI-22)}$$

Using the results of Garrick and Rubinow (1946):

$$\Omega_\theta X - \mu . r_\theta{}^2 + M_3 = 0 \qquad \text{(VI-23)}$$

$$M_4 + g\Omega_\theta X = 0 \qquad \text{(VI-24)}$$

In order to solve the single degree of freedom problem it is necessary to obtain values of M_4 as a function of k until the second equation is satisfied. The value of M_3 for that k is then used in the first equation to determine if X is reasonable:

$$X = 1 - \frac{M_3}{\Omega_\alpha} > 0 \qquad \text{(VI-25)}$$

The non-dimensional flutter frequency is:

$$\frac{\omega_F}{\omega_\theta} = \frac{1}{\sqrt{X}} \qquad \text{(VI-26)}$$

Accordingly the speed is:

$$\frac{U_F}{\hat{c}\omega_\theta} = \frac{1}{k\sqrt{X}} \qquad \text{(VI-27)}$$

Aerodynamic instability (flutter) occurs when $M_4 \leq \mathbf{0}$ while the L, M coefficients are given by the following:

$$L = L_3 + iL_4 = -\frac{2}{k^2}\{\int_0^1 C(x,0^+)dx - \int_0^1 C(x,0^-)dx\} \qquad \text{(VI-28)}$$

L

$$M = M_3 + iM_4 = -\frac{4}{k^2}\{\int_0^1 C(x,0^+)(x-x_0)dx - \int_0^1 C(x,0^-)(x-x_0)dx\} \qquad \text{(VI-29)}$$

And defining the following quantity, the non dimensional pitching moment, the analysis can be greatly facilitate:

$$C_m = \theta_0[c_{m\theta} + ikc_{m\theta'}]e^{ikt} \qquad \text{(VI-30)}$$

$$c_{m\theta'} = -\frac{1}{2}kM_4 \tag{VI-31}$$

$$c_{m\theta} = -\frac{1}{2}k^2M_3 \tag{VI-32}$$

VII. CONCLUSIONS

In this thesis, Platzer's (1973) approach to analyze the aeroelastic stability of flat-plate cascade and of airfoils in wind tunnels was extended to the case of two airfoils in close proximity to each other. The airfoils could oscillate in pitch with an arbitrary phase angle relative to each other. The analysis is valid for supersonic flow and is based on the small perturbation theory.

Two cases needed to be distinguished, namely airfoils with supersonic leading edge locus versus subsonic leading edge locus. In the former case, the leading edge of the upper airfoils is located upstream of the Mach wedge generated by the lower airfoil. In the latter case, the upper airfoil is within the Mach wedge of the lower airfoil.

For the case of supersonic leading edge locus a general formula could be derived for the pitch damping of the lower (store) airfoil which gives the dependence on Mach number, pitch axis location, phase angle, and proximity of the two airfoils. For the special case of out-of-phase oscillation between the two airfoils the result agrees with that derived by Platzer and Chalkey (1972), equations (37). The results showed that interference has a strongly destabilizing effect. In addition, for the same special case Platzer's (1973) equation (4.2) could be verified.

Furthermore, the analysis of the subsonic leading edge problem was begun and the pressure distribution on both sides of the upper foil was derived. In addition, a method of characteristics procedure was described which could be used to compare results.

LIST OF REFERENCES

1. Garrick, I. E., and Rubinow, *"Flutter and Oscillating Air Force Calculations for an Airfoil in Two Dimensional Supersonic Flow"*, *Naca* Report, 846, 1946.

2. Lane, F., *"Supersonic Flow past an Oscillating Cascade with Supersonic Leading Edge Locus"*, Journal of the Aeronautical Sciences, v. 24, pp. 65-66, January 1957.

3. Lane, F., *"System Mode Shapes in the Flutter of Compressor Blades Rows"*, Journal of the Aeronautical Sciences, v. 23, pp. 54-66, January 1956.

4. Miles, J. W., *"The Compressible Flow past an Oscillating Airfoil in a Wind Tunnel "*, Journal of the Aeronautical Sciences, v. 23, pp. 671-678, July 1956.

5. Platzer, M. F., "Wind Tunnel Interference on Oscillating Airfoils in Low Supersonic Flow". Acta Mechanica 16, pp. 115-126, Springer–Verlag 1973.

6. Platzer, M. F., Pierce C.V.,*"Theoritical Investigation of Wind Tunnel Interference Effects upon Fluttering Panels and Airfoils in Low Supersonic Flow"*, Proc. of AIAA/ASME Structures, Structural Dynamics and Material Conference, pp.17-24, Denver, Colorado, 22 April 1970.

7. Sauer, R.,*"Elementary Theorie des Langssam Scwingeden Uberschallflugel"*, Zeitschrift fur Angewandte Mathematik and Physik, pp. 248-253, 1950.

8. Platzer, M. F., Chalkey H.G., *"Theoretical Investigation of Supersonic Cascade Flutter and Related Interference Problems "*, Proc. of AIAA/ASME Structures, Structural Dynamics and Material Conference, San Antonio, Texas, April 1972.

9. Fung, Y. C., *"An Introduction to the Theory of Aeroelasticity"*, Dover, 1996.

10. Drake, D., G., *"The Motion of an Oscillating Airfoil in a Compressible Free Jet"*, Journal of the Royal Aeronautical Society, pp. 621-623, September 1956.

11. Drake, D.G.,*"The Oscillating Two Dimensional Airfoil between Porous Wall"*, Aeronautical Quarterly, v. 8, pp. 226-239, 1957.

12. Garrick, I. E., *"Non steady Wing Characteristics"*, Section F of Aerodynamics Components of Aircraft at High Speeds, Princeton University Press, 1957.

13. Turner, C. D., "Study of the Effect of Store Aerodynamic on Wing/Store Flutter", Proc.of AIAA ASME ASCE AHS. Struc. Dynamic Material Conference, 6-10 April 1981, Atlanta, GA, New York, NY, pp. 343-351, Paper: 81-0604.

14. Pollock, S. J., Sotomayer, W. A. Hutsell, L. J., *"Evaluation of the Methods for Prediction and Prevention of Wing/Store Flutter"*, Proc.of AIAA ASME ASCE AHS Struc. Dynamic Material Conference 22nd, 6-10 April 1981, Atlanta, GA, AIAA (CP 812), New York, NY, pp. 362-372, Paper: 81-0606.

15. Yang, Zhi-Chun Zhao, Ling-Cheng," *Wing/Store Flutter Analysis of an Airfoil in Incompressible Flow"*, Journal of Aircraft, v.26, pp. 583-587, 6 January 1989.

16. Turner C.D.,*"Effect of Store Aerodynamics on Wing/Store Flutter"*, Journal of Aircraft, v.19, pp. 574-580, 7 July 1982.

17. Silva, Walter A. Bennett, Robert M.Yang, *"Application of Transonic Small Disturbance Theory to the Active Flexible Wing Model "*, Journal of Aircraft, v.32, pp. 16-22, January 1995.

18. Peloubet R. P. Jr. Haller, R. L.,*"F-16 Active Flutter Suppression Program"*, Journal of Aircraft, v.26, pp. 583-587, 6 January 1989.

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center..2

   8725 John J. Kingman Rd., STE 0944

   Ft. Belvoir, Virginia 22060-6218

2. Dudley Knox Library..2

   Naval Postgraduate School

   411 Dyer Rd.

   Monterey, California, 93943-5101

3. Hellenic Air Force General Staff...1

   C1 Directorate

   Holargos, Athens, Greece